Income Inequality in General Equilibrium

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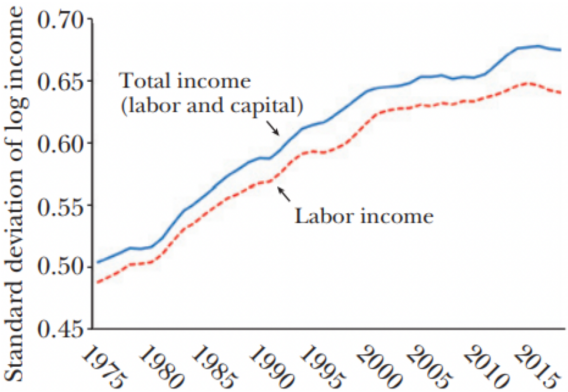
Ongoing research agenda on production networks

- **Firm heterogeneity**
  - The origins of firm heterogeneity: a production network approach (JPE, 2022)
  - Imperfect competition in firm-to-firm trade (JEEA, 2022)
  - The impact of firm-level polices on productivity growth and reallocation (R&R EER)
  - Price updating with production networks
  - Structural identification of productivity under biased technological change
  - Firm embeddings
- **Household heterogeneity**
  - Income inequality in general equilibrium
- **International trade**
  - Pecking order and core-periphery structure of international trade (RIE, 2020)
  - Multinational ownership and trade participation
  - Open Strategic Autonomy and inequality in the EU
- **Statistical classifications**
  - Correspondences of EU product classifications
  - World input-output tables with regional detail for Belgium

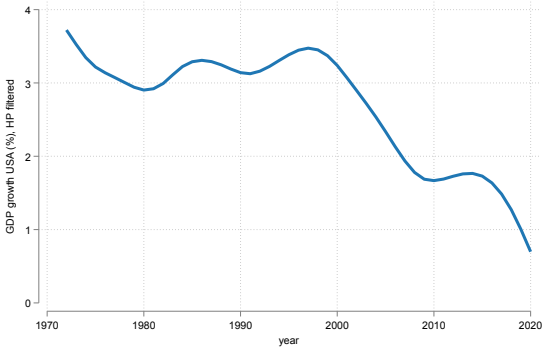
Motivation: US income inequality vs GDP growth

Figure: Income inequality



Notes: Sample of full-time/full-year workers age 25-64 earning $\geq$4$/hour (2018 dollars). Labor income: wages + salaries + net self employment. Capital income: incomes from interests + dividends + rents. Top 1% of the distribution trimmed (inconsistencies in collection very top over time). **Source:** Hoffman, Lee and Lemieux (JEP, 2020).

Figure: GDP growth



Notes: Yearly GDP growth in %. Hodrick-Prescott filtered, smoothing parameter $\lambda = 100$. **Source:** Global Economic Indicators, World Bank.

This paper

- **What is the relationship between income inequality and output growth?**
  - Growth accounting and welfare literatures are intrinsically related, but evolved separately
- **What is the role of labor mobility frictions in driving both outcomes?**
  - In growth accounting & decomposition?
  - In general equilibrium & counterfactuals?

What we do

- **Growth accounting**
  - Simple growth model with mobility frictions
  - Frictions generate wage inequality and hamper growth
  - Theil inequality index ("entropy") appears directly from economic setup
  - TFP is mis-measured if labor frictions exist (separate from other wedges)
- **General equilibrium**
  - Non-parametric model for GDP growth with labor frictions
  - Multi-sector, multi-factor model with input-output linkages
  - Labor mobility frictions $\rightarrow$ misallocation of workers $\rightarrow$ growth $\downarrow$, inequality $\uparrow$
  - General equilibrium effects: micro (wages, prices, quantities) and macro (output, inequality)
  - Full characterization up to 2nd order; 1st order for nested CES
- **Extensions**
  - Unemployment and minimum wages
  - Education frictions
- **Quantifying the effects of frictions on US GDP growth**
  - growth accounting and general equilibrium

Related literature

- **Growth accounting:** Solow (1957), Domar (1961), Kuznets (1961), Hulten (1978), Gollop et al., (1987), Basu-Fernald (2002), Petrin-Levinsohn (2012), Baqaee-Farhi (2019, 2020)
- **Growth and inequality:** Sen (1973), Aghion et al. (1999), Gabaix et al. (2016), Kaplan-Violante (2018)
- **Inequality measurement:** Lorenz (1905), Gini (1912), Theil (1967), Atkinson (1970), Shorrock (1984)
- **Misallocation:** Hsieh-Klenow (2009), Hsieh et al. (2019), Baqaee-Farhi (2020)
- **Sources of wage inequality:** Autor et al. (2003), Goldin-Katz (2007), Acemoglu-Autor (2011), Acemoglu-Restrepo (2020), Fortin-Lemieux-Lloyd (2021), Autor et al. (2014)
- **Economies with frictions:** Baqaee-Farhi (2020), Bigio-La'O (2020)
- **Envelope theorems:** Vickery (1961), Myerson (1981), Bulow-Klemperer (1996), Milgrom-Segal (2002)

Today

Simple growth model

General model

Comparative statics

Discussion

Quantitative application

Setup

- **Simplest growth model (e.g. Solow 1957)**
- **Aggregate output (real GDP)**

$$Y = zF\left(L_1, ..., L_N, K\right)$$

  - z: productivity shifter
  - $F\left(\cdot\right)$: production function with constant returns to scale
  - $L_1, ..., L_N$: workers allocated to labor type $i = 1, ..., N$ with $\sum_i L_i = L$
  - K: capital (perfectly mobile)
- **With prices**

$$P = \frac{1}{z} C\left(w_1, ..., w_N, r\right)$$

- **Mobility frictions across labor types**
  - Perfect mobility $\rightarrow$ all labor types receive the same wage $w_i = \bar{w}, \forall i$.
  - Perfect immobility $\rightarrow$ N fixed factors, each with its own wage w_i.
  - Imperfect mobility $\rightarrow$ frictions induce wage differentials $w_i - \bar{w}, \forall i$.

Social planner's problem

- **Maximize output subject to labor allocation frictions**

$$Y = \max_{\{L_i\}_{i \in N}} zF(\cdot) - \underbrace{\tau_L \left(\bar{L} - \sum_{i=1}^{N} L_i \right)}_{\text{Total labor constraint}} - \underbrace{\tau_K \left(\bar{K} - \sum_{i=1}^{N} K_i \right)}_{\text{Total capital constraint}} - \underbrace{\sum_{i=1}^{N} \mu_i \left(\bar{L}_i - L_i \right)}_{\text{Labor misallocation}}$$

  - $\bar{L}, \bar{K}$: total labor and capital supply (fixed) with Lagrange multipliers τ_L, τ_K
  - $\bar{L}_i$: imposed allocation for labor type i (e.g. not enough doctors)

- **Mobility constraints** $\mu_1, ..., \mu_N$
  - At the efficiency frontier: $\mu_i = 0$ and $\bar{w}$ is the unique wage, equal across all i
  - With frictions, $\mu_i = w_i - \bar{w}$, changes in imposed allocations can increase or decrease Y
  - μ_i are shadow prices of misallocation (change in GDP from change in constraint)

Growth accounting and inequality

- **GDP growth is** (first-order change around the initial steady state)

$$d\ln Y = \underbrace{d\ln z}_{\Delta\text{Productivity}} + \underbrace{\Lambda_L d\ln L + \Lambda_K d\ln K}_{\Delta\text{Factor supply}} - \underbrace{\Lambda_L \sum_{i=1}^{N} \left(\frac{L_i}{L}\right)\left(\frac{w_i}{\bar{w}}\right) d\ln\left(\frac{w_i}{\bar{w}}\right)}_{\Delta\text{Inequality}}$$

  $\Lambda_L \equiv \frac{\bar{w}L}{Y}$: labor income share; $\Lambda_K \equiv \frac{rK}{Y}$: capital share; $\bar{w} \equiv \frac{1}{L}\sum_i w_i L_i$: mean wage

- Δ inequality is a *first-order change* in the **Theil index** $\mathcal{I} \in [0, \infty)$, appears from economic principles

$$\mathcal{I} = -\sum_{i=1}^{N} \left(\frac{L_i}{L}\right)\left(\frac{w_i}{\bar{w}}\right)\ln\left(\frac{w_i}{\bar{w}}\right)$$

- **Maximizing growth is equal to minimizing inequality!**

TFP measurement

- **A typical exercise is to recover unobserved productivity $d \ln z$ from observables**

$$d \ln Y - \underbrace{\Lambda_L d \ln L - \Lambda_K d \ln K}_{\Delta\text{Factor supply}} = \underbrace{d \ln z}_{\Delta\text{Productivity}} + \underbrace{\Lambda_L \sum_{i=1}^{N} \left(\frac{L_i}{\bar{L}} \right) \left(\frac{w_i}{\bar{w}} \right) d \ln \left(\frac{w_i}{\bar{w}} \right)}_{\Delta\text{Inequality}}$$

- **Estimated productivity growth is biased with mobility frictions**
  - previous result son technology change do not hold if there exist mobility frictions
  - separate from other sources of TFP bias (inefficient economies, second order effects, ...)
  - extends to micro TFP as well... (other project)

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Overview

- **Dimensions**
  - Economy with multiple sectors and factors
  - Non-parametric: arbitrary elasticities of intermediates, factors, and skilled workers
- **Production**
  - Sectors use output from other sectors, labor and capital to produce their own output
  - Perfect competition with frictions
- **Households and labor**
  - Households are both consumers and workers
  - Workers face frictions to move across sectors
  - Inelastic labor supply (results with endogenous unemployment in paper)

Production and consumption

- **Dimensions:** $i \in N$ sectors and $f \in E$ labor types/occupations
- **Households/consumers** with identical homothetic preferences

$$\mathcal{Y} = C\left(\{c_i\}_{i \in N}\right)$$

subject to their budget constraint $\sum_{i \in N} p_i c_i = w_{if} L_{if}$ for each $i \in N$, $f \in E$

- **Output** for sector i, with constant returns to scale

$$y_i = F_i\left(z_{if}, \{L_{if}\}_{f \in E}, K_i, \{x_{ij}\}_{j \in N}\right)$$

- **Prices** for good/service i

$$p_i = C_i\left(z_{if}, \{w_{if}\}_{f \in E}, r, \{p_j\}_{j \in N}\right)$$

Workers

- **Workers of type f supply inelastic labor** in sector i based on preferences and wages
- **The share of workers of type f that choose to work in sector i** (Roy-Fréchet)

$$\Phi_{if} \equiv \frac{L_{if}}{L_f} = \frac{\phi_{if} w_{if}^{\kappa}}{\sum_{j=1}^{N} \phi_{jf} w_{jf}^{\kappa}}$$

  with ϕ_{if} location parameter (inverse mobility friction); κ dispersion parameter

- **Labor mobility**
  - $\kappa \to 0$ (perfect immobility): workers choose their allocation based on ϕ_{if} only
  - $\kappa \to \infty$ (perfect mobility): workers choose based on wages w_{if} only and $w_{if} = \bar{w}_f$ in GE
- **Wage gap:** observed $(\cdot)$ versus frictionless $(\cdot)^*$ labor allocations equals wage gaps

$$\Gamma_{if} \equiv \frac{(L_{if}/L_f)}{(L_{if}/L_f)^*} = \frac{w_{if}}{\bar{w}_f}$$

Key equilibrium relationships

- **Contribution of a sector i to nominal GDP** is its Domar weight

$$\lambda_i \equiv \frac{p_i y_i}{GDP} = \sum_{j=1}^{N} \underbrace{\frac{p_j c_j}{GDP}}_{\text{final consumption}} \underbrace{\Psi_{ji}}_{\text{Leontief inverse}}$$

$\rightarrow$ the importance of sector i as a direct and indirect supplier to final demand

  - $\sum_i \lambda_i > 1$ with intermediary goods (gap between gross output and value added)
  - Domar weights are endogenous here due to both production and consumption

- **Contribution of labor type f in sector i to nominal GDP**

$$\underbrace{\Lambda_{if}}_{\text{labor share in VA}} = \underbrace{\frac{w_{if} L_{if}}{p_i y_i}}_{\text{labor share in production}} \lambda_i$$

$\rightarrow$ the importance of labor f in sector i as direct and indirect supplier to final demand

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Overview

- **Series of potential shocks to the equilibrium outcome**
  - changes in sector-labor productivity z_{if}
  - mobility constraints κ
- **Their impact on**
  - micro: prices $\{p_i, w_{if}, r\}$ and quantities $\{L_{if}, K_i, x_{ij}\}$ for all $i, j \in N$ and $f \in E$
  - macro: real GDP $\mathcal{Y}$, inequality $\mathcal{I}$

Impact of productivity shocks z_{if} on GDP

- **First-order effect of Harrod-neutral productivity shocks z_{if} on real GDP**

$$\frac{d \ln \mathcal{Y}}{d \ln z_{if}} = \underbrace{\Lambda_{if}}_{\text{direct effect}} \underbrace{- \sum_j \sum_g \Lambda_{jg} \frac{d \ln \Gamma_{jg}}{d \ln z_{if}}}_{\text{inequality effect}}$$

- **Intuition**
  - direct effect (Hulten, 1978): Harrod-neutral shocks affect output $\propto$ income shares
  - inequality: productivity shocks change labor frictions Γ_{jg} in all sector-labor types

Impact of mobility shocks κ on GDP

- **First-order effect of shocks to mobility frictions κ on real GDP**

$$\frac{d \ln \mathcal{Y}}{d \ln \kappa} = \underbrace{- \sum_j \sum_g \Lambda_{jg} \frac{d \ln \Gamma_{jg}}{d \ln \kappa}}_{\text{inequality effect}}$$

- **Intuition**
  - No direct effect, only indirect effects through inequality (hence not in Hulten!)

- **Positive effect on GDP *only* if labor allocation improves**
  - If wages were initially higher in sectors that are more productive, GDP *increases*
  - Otherwise GDP *decreases*

Impact of productivity shocks z_{jg} on inequality

- **Effect of productivity shocks z_{jg} on wages w_{if}**

$$\frac{d \ln w_{if}}{d \ln z_{jg}} = \underbrace{\frac{d \ln GDP}{d \ln z_{jg}}}_{\text{aggregate channel}} + \underbrace{\frac{d \ln \Lambda_{if}}{d \ln z_{jg}}}_{\text{labor demand channel}} - \underbrace{\frac{d \ln L_{if}}{d \ln z_{jg}}}_{\text{labor supply channel}}$$

- **Aggregate channel**
  - impact of productivity changes on aggregate output
  - output shifter: no impact on inequality
  - this is the only channel in Cobb-Douglas economies

Impact of productivity shocks z_{jg} on inequality

- **Effect of productivity shocks z_{sg} on wages w_{if}**

$$\frac{d \ln w_{if}}{d \ln z_{jg}} = \underbrace{\frac{d \ln GDP}{d \ln z_{jg}}}_{\text{aggregate channel}} + \underbrace{\frac{d \ln \Lambda_{if}}{d \ln z_{jg}}}_{\text{labor demand channel}} - \underbrace{\frac{d \ln L_{if}}{d \ln z_{jg}}}_{\text{labor supply channel}}$$

- **Labor demand channel**
  - productivity shock induces changes in the value added share of other sector-labor types
  - split into *scale effect* (market size λ_i) and *substitution effect* (reallocation of factors Ω_{if})

Impact of productivity shocks on inequality

- **Effect of productivity shocks z_{sg} on wages w_{if}**

$$\frac{d \ln w_{if}}{d \ln z_{jg}} = \underbrace{\frac{d \ln GDP}{d \ln z_{jg}}}_{\text{aggregate channel}} + \underbrace{\frac{d \ln \Lambda_{if}}{d \ln z_{jg}}}_{\text{labor demand channel}} - \underbrace{\frac{d \ln L_{if}}{d \ln z_{jg}}}_{\text{labor supply channel}}$$

- **Labor supply channel**
  - productivity shocks induce workers to move across sectors
  - for $\kappa \to 0$: no reallocation of workers, inducing large effects on income inequality
  - for $\kappa \to \infty$: large reallocation, and no impact on wage inequality

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Parameterization

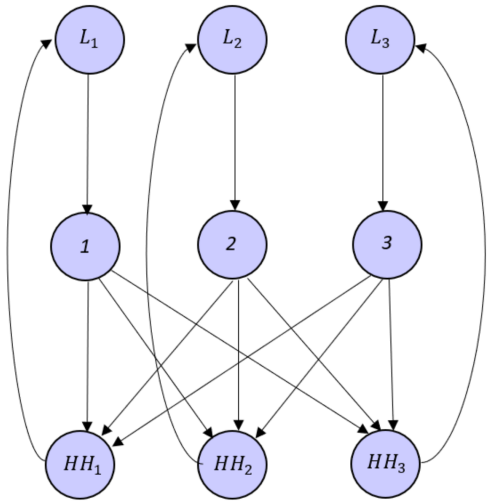
- **The contribution of the various channels depend on**
  - parameterization of the production/utility functions
  - the network structure of the economy
- **Examples with CES structures**
  - shutting down channels to gain intuition on total effects
- **Full characterization with nested CES**

Parameterization: Horizontal economy

Setup: multiple sectors, labor as only and fixed input
Result: only impact on inequality through sales shares

$$d \ln \frac{w_1}{w_2} = d \ln \frac{\lambda_1}{\lambda_2}$$

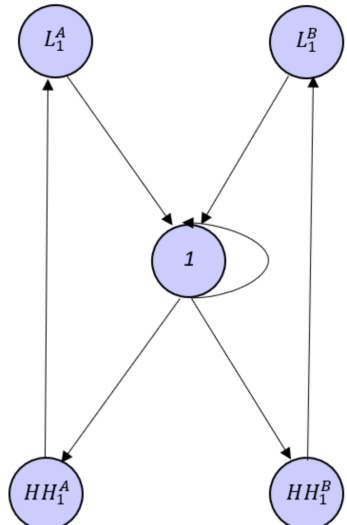
(cf. Acemoglu and Autor, 2011)



Parameterization: Roundabout economy

Setup: one sector, 2 labor types as input (skilled/unskilled), roundabout input use
Result: only impact on inequality through reallocation

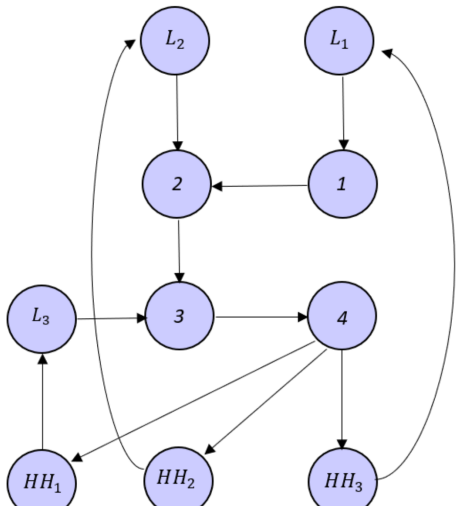
$$d \ln \frac{w_s}{w_u} = d \ln \frac{\Omega_{1s}}{\Omega_{1u}}$$



Parameterization: Vertical economy

Setup: multiple sectors, labor as input to each stage
Result: only impact on inequality through sales shares

$$d \ln \frac{w_1}{w_2} = d \ln \frac{\lambda_1}{\lambda_2} + d \ln \frac{\Omega_{1s}}{\Omega_{2u}}$$



Distance to the efficiency frontier

- **The economy is on the efficiency frontier if there is no income inequality**
- **How far are we from the frontier when income inequality exists?**
  - Following Hsieh and Klenow (2009) and Baqaee and Farhi (2020)
  - For small $d \ln \Gamma_{if}$

$$\mathcal{D} = \ln \mathcal{Y}(z, 1) - \ln \mathcal{Y}(z, \Gamma)$$

$$\approx \underbrace{\frac{1}{2} \sum_i \sum_f \Lambda_{if} \, d \ln \Gamma_{if} \, d \ln \Lambda_L}_{\text{Labor share}} - \underbrace{\frac{1}{2} \sum_i \sum_f \Lambda_{if} \, d \ln \Gamma_{if} \, d \ln \Lambda_{if}}_{\text{Reallocation}} + \underbrace{\frac{1}{2} \sum_i \sum_f \Lambda_{if} \left(d \ln \Gamma_{if} \right)^2}_{\text{Non-linearities}}$$

Extension 1: unemployment and minimum wages

- **Labor supply function**
  - workers supply labor following $\mathcal{U} = \mathcal{C} + \frac{(\bar{L}-L)^{1-\varphi}}{1-\varphi} - \frac{\underline{w}}{P_c} L$
    with $\underline{w}$ is (exogenous) minimum wage and φ is the labor supply utility parameter (disutility from work)
- **Unemployment**
  - the level of unemployment is given by $u \equiv \bar{L} - L = \left(\frac{\bar{w}}{P_c} - \frac{\underline{w}}{P_c}\right)^{-\frac{1}{\varphi}}$
  - unemployment decreases in the gap between mean wage and minimum wage
- **Impact of shocks (e.g. productivity)**

$$\frac{d \ln \mathcal{Y}}{d \ln z_{if}} = \underbrace{\Lambda_{if}}_{\text{direct effect}} + \underbrace{\Lambda_L \frac{d \ln L}{d \ln z_{if}}}_{\text{indirect effect}} - \underbrace{\sum_j \sum_g \Lambda_{jg} \frac{d \ln \Gamma_{jg}}{d \ln z_{if}}}_{\text{inequality effect}}$$

Extension 2: mobility across education levels

- **Mobility of workers across education levels** κ_e
  - Workers *ex ante* choose a tuple {sector, labor type, education} that maximizes their expected wages given frictions
  - Step 1: workers choose an education level based on the wage distribution for each education level
  - Step 2: choose sector-occupation based on the marginal distribution of wages for a given educuation level
  - All weighted by additional friction parameters of the Frechet κ_e

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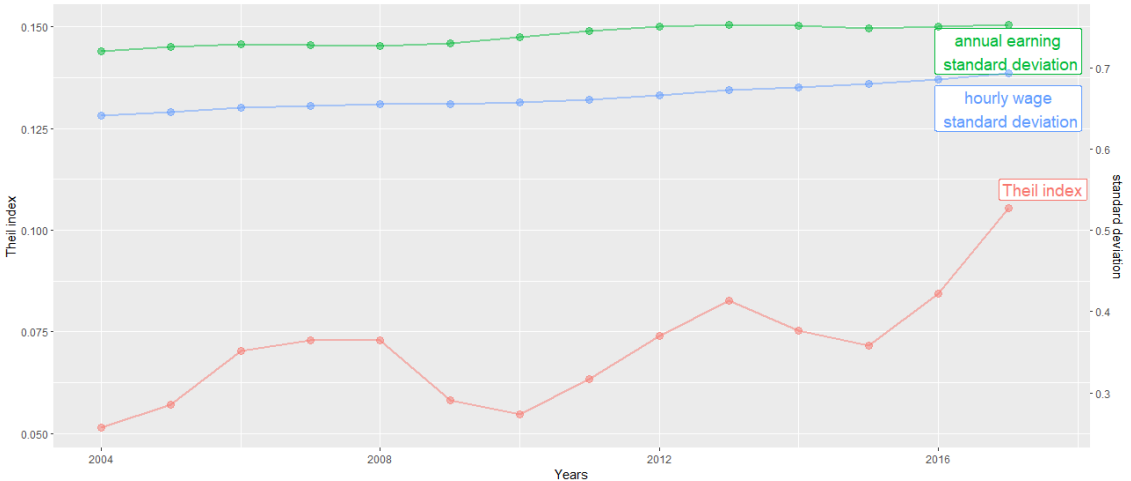
Data

- **Annual Social and Economic Supplement (US Census Bureau)**
  - annual earning and hourly wages
  - hours and weeks worked
  - 156 sectors matched between CPS industries and LBS IO data
  - 604 occupations in total
  - from 2003 to 2018
  - keep full-time full-year workers

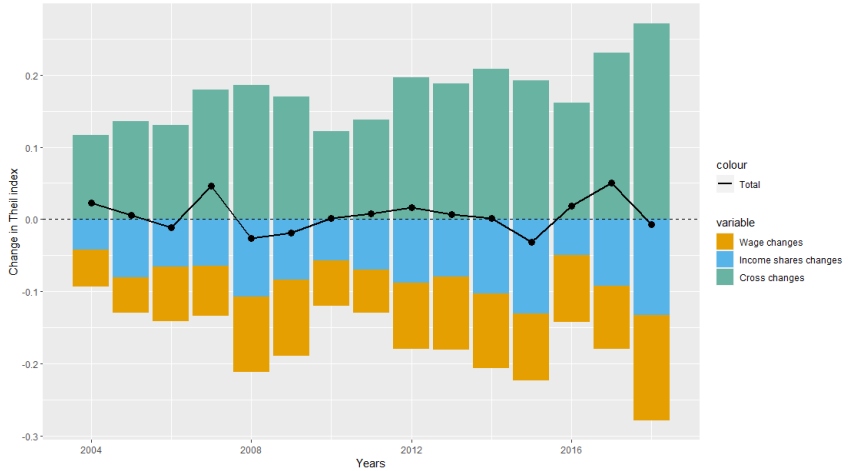
- **Input-Output tables (Bureau of Labor Statistics)**
  - intersectoral linkages between 156 sectors
  - gross output and value-added by sector

- **Growth accounting data (Federal Reserve Board SF)**
  - Growth in production, hours worked, capital used and TFP
  - 2003-2018

Income inequality



Theil index changes, decomposition

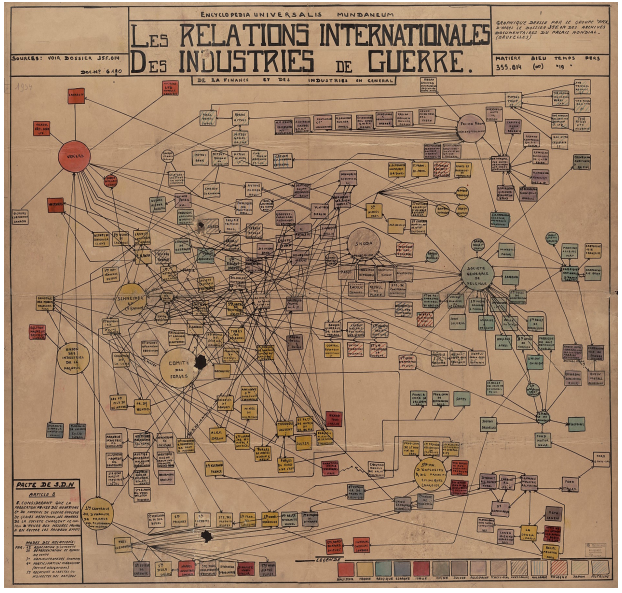


$$d\mathcal{I} = \underbrace{\sum_i \sum_f \Lambda_{fi,t-1} d \ln \left(\frac{w_{fi,t}}{\bar{w}_t} \right)}_{\text{Wage changes}} + \underbrace{\sum_i \sum_f d\Lambda_{fi,t} \ln \left(\frac{w_{fi,t-1}}{\bar{w}_{t-1}} \right)}_{\text{Income shares changes}} + \underbrace{\sum_i \sum_f d\Lambda_{fi,t} d \ln \left(\frac{w_{fi,t}}{\bar{w}_t} \right)}_{\text{Crosschanges}}$$

Conclusion

- **Income inequality is bad for economic growth**
  - This paper: due to misallocation of workers across occupations
  - Theil index as inequality measure from economic principles
- **Provide a general quantitative framework**
  - Non-parametric model of output growth with frictions
  - Multi-sector, multi-factor with IO linkages
  - Impact of productivity shocks and labor frictions on wage inequality and aggregate growth
- **Policy implications**
  - Maximizing output and minimizing inequality are the same
  - Possible to calculate social marginal value of policies

Thank you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