A Theory of Equality and Growth

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Motivation

- **Workers are imperfectly mobile across jobs**
  - E.g. due to education, training, information asymmetry, ...
  - Workers do not move only following wages.
- **What is the role of mobility frictions for aggregate growth and wage inequality?**
  - Growth accounting: either perfectly mobile or perfectly immobile.
  - General equilibrium: channels of wage inequality and growth?

What we do

- **Develop non-parametric growth accounting with mobility frictions**
  - Think Solow with imperfect labor mobility.
  - Provide an envelope theorem for aggregate growth with mobility frictions.
  - Impact of frictions on aggregate growth is exactly a change in the Theil index.
- **Quantify the impact of frictions on aggregate growth and wage inequality**
  - General equilibrium framework with labor mobility frictions.
  - Multiple sectors, occupations, factors with input-output linkages.
  - General non-parametric results and parametric economies.
  - GE effects: micro (wages, prices, quantities) and macro (output, wage inequality)
- **Application to US labor markets, growth and inequality**
  - (not today)

Today

Introduction

Aggregate growth with mobility frictions

General equilibrium: setup

General equilibrium: non-parametric comparative statics

General equilibrium: parametric intuition

Setup

Consider a **general aggregate production function** with labor (L) and capital (K), where each HH supplies its labor to different tasks $t \in \mathcal{T}$

$$Y = zF\left(l_1, ..., l_{\mathcal{T}}, \bar{K}\right) \text{ with } \sum_{t \in \mathcal{T}} l_t = \bar{L}$$

where

- Y is real GDP, z is a Hicks-neutral productivity shifter,
- $F(\cdot)$ is a non-parametric production function with constant returns to scale,
- "Tasks" t are generic for now (e.g. occupation, skills, requiring a particular degree).

An envelope theorem with frictions

Solve the social planner's problem with restrictions to the workers' allocation across tasks

$$Y\left(z, \bar{L}, \bar{l}_1, ..., \bar{l}_{\mathcal{T}}, \bar{K}\right) = \max_{\{l_t\}_{t \in \mathcal{T}}} z\, F(l_1, ..., l_{\mathcal{T}}, K) - \tau_K \left(\bar{K} - K\right) - \sum_{t \in \mathcal{T}} \mu_t \left(\bar{l}_t - l_t\right)$$

where

- $\bar{l}_t$ is the imposed labor allocation,
- $\frac{\partial F}{\partial l_t} = \mu_t = w_t$ in a competitive equilibrium,
- τ_K and the vector μ_t are the lagrange multipliers of capital and mobility constraints.

An envelope theorem with frictions (cont'd)

First-order change around the initial equilibrium

$$d\ln Y = d\ln z + \left(\frac{rK}{GDP}\right) d\ln K + \sum_{t\in\mathcal{T}} \left(\frac{w_t l_t}{GDP}\right) d\ln \bar{l}_t$$

Using Shephard's lemma and plugging in wages

$$d\ln Y = \underbrace{d\ln z}_{\Delta\text{Technology}} + \underbrace{\left(\frac{rK}{GDP}\right) d\ln K}_{\Delta\text{Capital supply}} + \underbrace{\left(\frac{\tilde{w}L}{GDP}\right) d\ln L}_{\Delta\text{Labor supply}} - \underbrace{\left(\frac{\tilde{w}L}{GDP}\right) \sum_{t\in\mathcal{T}} \left(\frac{l_t}{L}\right) \left(\frac{w_t}{\tilde{w}}\right) d\ln \left(\frac{w_t}{\tilde{w}}\right)}_{\Delta\text{Inequality}}$$

where $\tilde{w} = \sum_{t\in\mathcal{T}} \frac{w_t l_t}{L}$ is the average wage in the economy.

An envelope theorem with frictions (cont'd)

$$d\ln Y = \underbrace{d\ln z}_{\Delta\text{Technology}} + \underbrace{\left(\frac{rK}{GDP}\right) d\ln K}_{\Delta\text{Capital supply}} + \underbrace{\left(\frac{\tilde{w}L}{GDP}\right) d\ln L}_{\Delta\text{Labor supply}} - \underbrace{\left(\frac{\tilde{w}L}{GDP}\right) \sum_{t\in\mathcal{T}} \left(\frac{l_t}{L}\right) \left(\frac{w_t}{\tilde{w}}\right) d\ln\left(\frac{w_t}{\tilde{w}}\right)}_{\Delta\text{Inequality}}$$

Intuition: an increase in the wedge $(w_t/\tilde{w})$ between task-specific marginal productivity w_t and the average marginal productivity $\tilde{w}$ reflects an increasing misallocation of workers.

Discussion

- Δ Inequality maps directly to the change in workers' allocation across tasks, l_t/L.
- If ΔInequality$= 0$, we're back to Solow.
- If ΔInequality> 0, $d\ln Y \downarrow$.

Wage inequality as a Theil index

Definition of the Theil Index $I(\Theta)$:

$$I(\Theta) = \underbrace{Q(\mathcal{H})}_{\text{maximum entropy}} - \underbrace{Q(\Theta)}_{\text{current entropy}}$$

where $\Theta_{t,L} = \frac{w_t l_t}{\tilde{w} L}$ are the labor income shares for task t; Θ is the distribution of $\Theta_{t,L}$.

Plugging in labor income shares $\frac{w_t l_t}{\tilde{w} L}$

$$\begin{aligned} I &= \sum_{t \in \mathcal{T}} \left(\frac{1}{\bar{L}}\right) \ln\left(\frac{\bar{L}}{1}\right) - \sum_{t \in \mathcal{T}} l_t \left(\frac{1}{\bar{L}} \frac{w_t}{\tilde{w}}\right) \ln\left(\frac{\bar{L}}{1} \frac{\tilde{w}}{w_t}\right) \\ &= \sum_{t \in \mathcal{T}} \left(\frac{l_t}{L}\right) \left(\frac{w_t}{\tilde{w}}\right) \ln\left(\frac{w_t}{\tilde{w}}\right) \end{aligned}$$

The Theil index is the entropy of wage ratios with labor income shares as probability weights.

Aggregate growth frictions as a change in the Theil index

A change in the Theil index is then (keeping income shares weights fixed)

$$
\begin{aligned}
d\, I(\Theta) &= d\, Q(\mathcal{H}) - d\, Q(\Theta) \\
&= \sum_{t \in \mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L} \right) d \ln \bar{L} - \sum_{t \in \mathcal{T}} w_t l_t d \ln \left(\frac{\bar{L}}{1} \frac{\tilde{w}}{w_t} \right) \\
&= \sum_{t \in \mathcal{T}} \left(\frac{w_t l_t}{\tilde{w} L} \right) d \ln \left(\frac{w_t}{\tilde{w}} \right)
\end{aligned}
$$

Key result 1: The previously *ad hoc* Theil index arrives naturally from simple growth theory!

Key result 2: $d \ln Y / d\, I(\Theta) = -\Theta_L$ is the semi-elasticity of real GDP with respect to income inequality where $\Theta_L = \tilde{w} L / GDP$ is the total labor income share.

Discussion 1: Aggregate misallocation

Relationship to Baqaee and Farhi (2020)

Mobility frictions are different and independent from their misallocation measure which arrive from 'wedges' on prices in the goods markets

$$d \ln Y = \underbrace{d \ln z}_{\text{Technology}} + \underbrace{\grave{\Theta}_L d \ln L + \grave{\Theta}_K d \ln K}_{\text{Factor supply}} \underbrace{- \sum_{t \in \mathcal{T}} \grave{\Theta}_{t,L} d \ln \Theta_{t,L}}_{\text{Allocative efficiency}} \underbrace{- \grave{\Theta}_L \sum_{t \in \mathcal{T}} \left(\frac{\grave{\Theta}_{t,L}}{\grave{\Theta}_L} \right) d \ln \left(\frac{w_t}{\tilde{w}} \right)}_{\text{Inequality}}$$

where the $\grave{\Theta}$ are the markup-corrected income shares of the different HH.

Discussion 2: Cost-benefit analysis

What are the gains from removing frictions?
We provide a measure Δ of the distance to the efficiency frontier (the output point where the wage ratio $w_t/\tilde{w} = 1, \forall t$):

$$\Delta = \ln\left(\frac{Y^*}{Y}\right) \approx \Theta_L \sum_i \left(\frac{\Theta_{t,L}}{\Theta_L}\right) \ln\left(\frac{w_t}{\tilde{w}}\right) = \Theta_L \times I(\Theta)$$

Again, the Theil index $I(\Theta)$ appears naturally, as a mapping for Δ.

What is the maximum cost to remove all wage inequality while keeping Y fixed?
Suppose a government levies a lump-sum tax τ to fund the removal of frictions at a government spendings G. Then the maximum cost is just

$$dY > G \iff d\ln Y > G/Y = \tau Y$$

This is actually a lower bound, as τ is used only to remove frictions and thus thrown away.

Today

Introduction

Aggregate growth with mobility frictions

General equilibrium: setup

General equilibrium: non-parametric comparative statics

General equilibrium: parametric intuition

Setup

Context

- From growth accounting, we know how to measure efficiency losses from labor frictions.
- But what are the channels through which aggregate losses and inequality can occur?

General equilibrim framework

- Economy with multiple sectors, occupations, and factors.
- Arbitrary elasticities of intermediates, factors, and occupations.

Production

- Sectors combine factors with inputs from other sectors to produce own output.
- Perfect competition with labor mobility frictions.

Households and labor

- Households are both consumers and workers.
- Workers face frictions to move across sectors.

Production

- **Dimensions:** $o \in \mathcal{O}$ labor types/occupations and $i \in \mathcal{N}$ sectors.
- **Output** for sector i, with constant returns to scale:

$$y_i = F_i\Big(\underbrace{\{z_{io,L}\}_{o\in\mathcal{O}}, z_K}_{\text{input-specific shifters}}, \underbrace{\{l_{io}\}_{o\in\mathcal{O}}, k_i}_{\text{factor quantities}}, \underbrace{\{x_{ij}\}_{j\in\mathcal{N}}}_{\text{input quantities}} \Big)$$

- **Pricing** for output i at marginal costs:

$$p_i = f_i\left(\{z_{io,L}\}_{o\in\mathcal{O}}, z_K, \{w_{io}\}_{o\in\mathcal{O}}, r, \{p_j\}_{j\in\mathcal{N}}\right)$$

Labor allocation

Workers are allocated across occupations o by maximizing a CRS allocation function:

$$L = G\left(\nu_1 l_1, \cdots, \nu_{\mathcal{O}} l_{\mathcal{O}}\right)$$

where ν_o is the mobility cost for occupation o; l_o is the labor allocated to o.

Within each occupation, workers are allocated across sectors following $G_o(\cdot)$

$$l_o = G_o\left(\phi_{1o} l_{1o}, \cdots, \phi_{\mathcal{N}o} l_{\mathcal{N}o}\right)$$

where ϕ_{io} is the mobility cost for o in i; l_{io} is the labor allocated to o in sector i.

Maximizing $G(\cdot)$ and $G_o(\cdot)$ wrt. the HH budget constraint, gives the **optimal allocation** across occupations and sectors as a function of wages and mobility costs:

$$\left(\frac{l_{1o}}{l_o}\right) = g_o\left(\phi_{1o} w_{1o}, \cdots, \phi_{\mathcal{N}o} w_{\mathcal{N}o}\right); \qquad \left(\frac{l_o}{L}\right) = g\left(\nu_1 w_1, \cdots, \nu_{\mathcal{O}} w_{\mathcal{O}}\right)$$

which gives the following joint allocation per sector and occupation:

$$\left(\frac{l_{io}}{L}\right) = \left(\frac{l_{io}}{l_o}\right) \times \left(\frac{l_o}{L}\right) = g_o\left(\phi_{1o} w_{1o}, \cdots, \phi_{\mathcal{N}o} w_{\mathcal{N}o}\right) \times g\left(\nu_1 w_1, \cdots, \nu_{\mathcal{O}} w_{\mathcal{O}}\right)$$

Utility and consumption

Total labor supply is determined by an aggregate utility function

$$\mathscr{U} = C - h(L)$$

where C is consumption and $h(L)$ reflects the disutility from working.

On the consumption side, households share identical homothetic preferences:

$$Y = C(c_1, \cdots, c_{\mathcal{N}})$$

where total income of the group of workers in occupation o and sector i is equal to $w_{io} l_{io}$, which pins down the budget constraint of each household.

Input-output definitions

Matrix elements $j \rightarrow i$

Tech coeffs: $\Omega_{ij,I} = \frac{p_j x_{ij}}{p_i y_i}$

Leontief inverse: $\Psi = (I - \Omega_I)^{-1}$

Labor share: $\Omega_{io,L} = \frac{w_{io} l_{io}}{p_i y_i}$

Capital share: $\Omega_{i,K} = \frac{r k_i}{p_i y_i}$

Final demand: $\Upsilon_i = \frac{p_i c_i}{GDP}$

Income share HH: $\Theta_{io,L} = \frac{w_{io} l_{io}}{GDP}$

		Use			Final demand
		1	$\cdots$	$\mathcal{N}$	
Supply	1	$\Omega_{11,I}$	$\cdots$	$\Omega_{\mathcal{N}1,I}$	Υ_i
	$\vdots$	$\vdots$	$\Omega_{ij,I}$	$\vdots$	$\vdots$
	$\mathcal{N}$	$\Omega_{1\mathcal{N},I}$	$\cdots$	$\Omega_{\mathcal{N}\mathcal{N},I}$	$\Upsilon_{\mathcal{N}}$
Labor	1	$\Omega_{11,L}$	$\cdots$	$\Omega_{\mathcal{N}1,L}$	
	$\vdots$	$\vdots$	$\Omega_{io,L}$	$\vdots$	
	$\mathcal{N} \times \mathcal{O}$	$\Omega_{1\mathcal{O},L}$	$\cdots$	$\Omega_{\mathcal{N}\mathcal{O},L}$	
Capital		$\Omega_{1,K}$	$\cdots$	$\Omega_{\mathcal{N},K}$	

The importance of sectors and HH in real GDP

Real GDP: $Y = C$

Supply centrality: The importance of sectors and HH for production is given by their *direct* and *indirect* importance in the production of final demand:

Sectors: through their Domar weights ϑ_i:

$$\vartheta_i \equiv \frac{p_i y_i}{GDP} = \sum_j \underbrace{\Upsilon_j}_{\textbf{FD share}} \underbrace{\Psi_{ji}}_{\textbf{Leontief inverse}}$$

$\rightarrow$ the importance of sector i as direct and indirect supplier to final demand.

Households: through their labor income shares $\Theta_{io,L}$:

$$\Theta_{io,L} \equiv \frac{w_{io} l_{io}}{GDP} = \sum_j \underbrace{\Upsilon_j}_{\text{FD share}} \underbrace{\Psi_{ji}}_{\text{Leontief inverse}} \underbrace{\Omega_{io,L}}_{\text{labor share}}$$

$\rightarrow$ the importance of labor in o in sector i as direct and indirect supplier to final demand.

The importance of sectors and HH in real GDP (cont'd)

Generalized Domar weights: Importance of a sector i to any other downstream sector m directly and indirectly through any j

$$\theta_{mi} = \sum_j \Omega_{mj,I} \Psi_{ji}$$

- i.e. θ_{mi} is a Domar weight, where the final demand is replaced by any sector m.

Demand centrality: Importance of a sector as a buyer to other sectors, and demand of labor directly and indirectly through any j

$$\xi_m = \sum_j \Psi_{mj} \qquad \text{(Leontief multiplier)}$$

$$\Xi_{mo,L} = \sum_j \Psi_{mj} \Omega_{jo,L} \qquad \text{(multiplier for labor demand } o \text{ in use of } m\text{)}$$

Prices

By Sheppard's lemma, we have that price changes are functions of the centrality measures:

$$d\ln p_i = \sum_j \sum_o \Xi_{ioL,j} d\ln\left(\frac{w_{jo}}{z_{jo,L}}\right) + \sum_j \Xi_{iK,j} d\ln r$$

Intuition: when a factor price w_{jo} changes, it impacts sector i's price p_i through i's direct and indirect importance as a *buyer* of that factor through $\Xi_{ioL,j}$.

Equilibrium

General equilibrium is given by:

- vectors of prices $\{p_i\} \forall i \in \mathcal{N}$, wages $\{w_{io}\} \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$, a price of capital r,
- vectors of intermediate goods $\{x_{ij}\} \forall i, j \in \mathcal{N}$,
- vectors of capital use $\{k_i\} \forall i \in \mathcal{N}$, and labor use $\{l_{io}\} \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$,
- vectors of output $\{y_i\} \forall i \in \mathcal{N}$, and a vector of consumption levels $\{c_i\} \forall i \in \mathcal{N}$.

Which are determined jointly by the following conditions:

1. Firms maximize profits, equal to zero
2. Workers maximize their utility
3. Labor, capital and goods markets clear:

$$L = \sum_o \sum_i l_{io}; \qquad \bar{K} = \sum_i k_i; \qquad y_i = c_i + \sum_j x_{ji}$$

For a vector of productivity shocks $\{z_{io,L}\} \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$ and **mobility costs** $\{\nu_o\} \forall o \in \mathcal{O}$ and $\{\phi_{io}\} \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$.

Today

Introduction

Aggregate growth with mobility frictions

General equilibrium: setup

General equilibrium: non-parametric comparative statics

General equilibrium: parametric intuition

Overview

- **Impact of shocks to equilibrium outcomes**
  - Sector-labor productivity $\{z_{io,L}\}$.
  - Mobility constraints $\{\nu_o\}$ (occupation) and $\{\phi_{io}\}$ (sector-occupation).
- **Their impact on**
  - Macro: real GDP Y, inequality $\mathcal{I}$.
  - Micro: prices $\{p_i,\ w_{io},\ r\}$ and quantities $\{l_{io}\ , k_i,\ x_{ij}\}$ for all $i, j \in \mathcal{N}$ and $o \in \mathcal{O}$.

Shock 1: Impact of productivity shocks on aggregate growth

Up to first order, the impact of an input-specific productivity shock on growth is given by:

$$d \ln Y = \underbrace{\sum_i \sum_o \Theta_{io,L} d \ln z_{io,L}}_{\text{direct effect}} + \underbrace{\sum_i \sum_o \Theta_L \frac{d \ln L}{d \ln z_{io,L}} d \ln z_{io,L}}_{\text{aggregate labor supply}} - \underbrace{\sum_i \sum_o \sum_j \sum_p \Theta_{jp,L} \frac{d \ln(w_{jp}/\tilde{w})}{d \ln z_{io,L}} d \ln z_{io,L}}_{\text{worker reallocation}}$$

Intuition

- **Direct effect** (Hulten, 1978): a Harrod-neutral productivity shock has an impact on real GDP proportional to the income share of this labor type, $\Theta_{io,L}$.
- **Indirect effects:** Real GDP effects through the change in aggregate labor supply and the reallocation of workers across sectors and occupations following endogenous changes in wages, even with fixed frictions.

Shock 2: Impact of mobility cost shocks on aggregate growth

Up to first order, the impact of a mobility cost shock on growth is given by:

$$d\ln Y = \underbrace{\sum_i \sum_o \Theta_{io,L} d\ln\phi_{io} + \sum_o \Theta_{o,L} d\ln\nu_o}_{\text{direct effect}} + \underbrace{\sum_i \sum_o \Theta_L \frac{d\ln L}{d\ln z_{io,L}} d\ln\phi_{io} + \sum_o \Theta_L \frac{d\ln L}{d\ln\nu_o} d\ln\nu_o}_{\text{aggregate labor supply}}$$
$$\underbrace{- \left(\sum_i \sum_o \sum_j \sum_p \Theta_{jp,L} \frac{d\ln(w_{jp}/\tilde{w})}{d\ln\phi_{io}} d\ln\phi_{io} + \sum_o \sum_j \sum_p \Theta_{jp,L} \frac{d\ln(w_{jp}/\tilde{w})}{d\ln\nu_o} d\ln\nu_o \right)}_{\text{worker reallocation}}$$

Intuition:

- Direct effects: change in labor allocation from changes in frictions.
- Indirect effects: changes in aggregate labor supply and reallocation of workers from wage changes.

Shock 3: Impact of productivity shocks on wage inequality

Effect of input-specific productivity shocks $z_{sp,L}$, on wages w_{io}:

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d\ln l_{io}}{d\ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d\ln GDP}{d\ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

$\rightarrow$ consider all channels in detail (plus additional intuition in parametric results below).

Shock 3: Impact of productivity shocks on wage inequality

$$\frac{d \ln w_{io}}{d \ln z_{sp,L}} = \underbrace{\frac{d \ln \Theta_{io,L}}{d \ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d \ln l_{io}}{d \ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d \ln GDP}{d \ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Labor demand channel

$$\frac{d \ln \Theta_{io,L}}{d \ln z_{sp,L}} = \underbrace{\frac{d \ln \Omega_{io,L}}{d \ln z_{sp,L}}}_{\textbf{labor use effect}} + \underbrace{\frac{d \ln \vartheta_i}{d \ln z_{sp,L}}}_{\textbf{scale effect}}$$

- Change in income shares $\Theta_{io,L}$ from a change in **labor use** $\Omega_{io,L} = \frac{w_{io} l_{io}}{p_i y_i}$ and/or a change in market **size** $d \ln \vartheta_i$.
- **Intuition**: New equilibrium prices and wages. Firms and consumers update choices and new labor shares and sector sizes result in equilibrium.

Shock 3: Impact of productivity shocks on wage inequality

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d\ln l_{io}}{d\ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d\ln GDP}{d\ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Labor supply channel

$$\frac{d\ln l_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln(l_{io}/l_i)}{d\ln z_{sp,L}}}_{\text{sectoral share}} + \underbrace{\frac{d\ln(l_o/L)}{d\ln z_{sp,L}}}_{\text{occupational share}} + \underbrace{\frac{d\ln L}{d\ln z_{sp,L}}}_{\text{Total labor supply}}$$

- Productivity shocks induce workers to move across sectors (with fixed ν and ϕ).
- Perfect immobility: no reallocation of workers, inducing large effects on income inequality.
- Perfect mobility: large reallocation, no wage inequality.

Shock 3: Impact of productivity shocks on wage inequality

Effect of input-specific productivity shocks $z_{sp,L}$, on wages w_{io}:

$$\frac{d\ln w_{io}}{d\ln z_{sp,L}} = \underbrace{\frac{d\ln \Theta_{io,L}}{d\ln z_{sp,L}}}_{\text{Labor demand channel}} - \underbrace{\frac{d\ln l_{io}}{d\ln z_{sp,L}}}_{\text{Labor supply channel}} + \underbrace{\frac{d\ln GDP}{d\ln z_{sp,L}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}, \forall o \in \mathcal{O}$$

Aggregate channel

$$\frac{d\ln GDP}{d\ln z_{sp,L}} = \underbrace{\Theta_{sp,L}}_{\text{Productivity}} + \underbrace{\Theta_L \frac{d\ln L}{d\ln z_{sp,L}}}_{\text{Total labor supply}} \underbrace{- \Theta_L \sum_j \sum_o \left(\frac{\Theta_{jo,L}}{\Theta_L}\right) \frac{d\ln \Gamma_{jo}}{d\ln z_{sp,L}}}_{\text{Inequality}}$$

- Impact of productivity changes on aggregate output.
- Output shifter: no impact on inequality.
- This is the only channel in Cobb-Douglas economies.

Today

Introduction

Aggregate growth with mobility frictions

General equilibrium: setup

General equilibrium: non-parametric comparative statics

General equilibrium: parametric intuition

Parametric form

Production: firms use a sector-specific nested CES technology

$$y_i = \left(\omega_{i,L}^{\frac{1}{\sigma}} l_i^{\frac{\sigma-1}{\sigma}} + \omega_{i,K}^{\frac{1}{\sigma}} k_i^{\frac{\sigma-1}{\sigma}} + \sum_j \omega_{ij}^{\frac{1}{\sigma}} x_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution between factors and intermediate goods.

Consumption: households have the same preferences across goods:

$$\mathscr{U}_c = \left(\sum_j \upsilon_j^{\frac{1}{\rho}} c_j^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

where υ_j is a preference shifter, and ρ is the elasticity of substitution across goods.

Parametric form

Worker allocation: each household h draws a vector of idiosyncratic frictions φ_{io}^{h}, drawn from a nested joint Fréchet distribution G:

$$G \sim \exp\left(\sum_{o\in\mathcal{O}} \nu_o \left(\sum_{i\in\mathcal{N}} \phi_{io} \left(\varphi_{io}\right)^{-\kappa}\right)^{\frac{\lambda}{\kappa}}\right)$$

with upper tier Frechet (ν, λ) and lower tier (ϕ, κ) location and dispersion parameters.

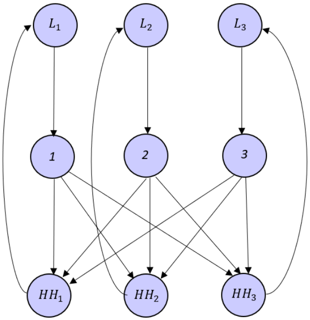
In equilibrium, the **share of workers specializing in occupation** o is given by:

$$\Phi_o = \frac{l_o}{L} = \frac{\nu_o w_o^{\lambda}}{w^{\lambda}}$$

where $w = \left(\sum_o \nu_o w_o^{\lambda}\right)^{\frac{1}{\lambda}}$ is the wage index of workers.
The **share of workers with occupation** o **supplying their labor to sector** i is equal to:$\Phi_{io} = \frac{l_{io}}{l_o} = \frac{\phi_{io} w_{io}^{\kappa}}{w_o^{\kappa}}$ where $l_o = \sum_i l_{io}$ and $w_o = \left(\sum_j \phi_{jo} w_{jo}^{\kappa}\right)^{\frac{1}{\kappa}}$.

Horizontal economy

Setup: Each HH supplies its labor to one sector i. Each sector only uses labor as input. Output sold directly to final demand.

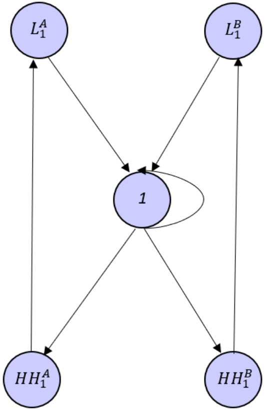


- No intermediate goods→sales shares and income shares coincide ($\vartheta_i = \Theta_i$). HH h that supplies labor in sector i is the only one affected by chanegs in sales of i.
- Impact on wage inequality depends on FD elasticity ρ and how HH reallocate expenditures across sectors.
- Only a scale effect, no substitution effect

$$d \ln \left(\frac{w_1}{w_2} \right) = d \ln \left(\frac{\vartheta_1}{\vartheta_2} \right) = d \ln \left(\frac{\Upsilon_1}{\Upsilon_2} \right)$$
$$= \left(\frac{\rho - 1}{\rho} \right) d \ln \left(\frac{z_{1,L}}{z_{2,L}} \right)$$

Roundabout economy

Setup: Each HH supplies one labor type *o* to only one sector in the economy. This sector's output is sold to itself and to final consumers.

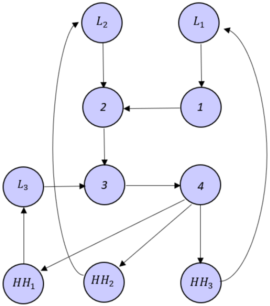


- Two labor types.
- Impact on inequality depends only on factor elasticity of substitution.
- Only a substitution effect, no scale effect

$$d\ln\left(\frac{w_{1s}}{w_{1u}}\right) = d\ln\left(\frac{\Omega_{1s,L}}{\Omega_{1u,L}}\right)$$
$$= \left(\frac{\sigma-1}{\sigma}\right) d\ln\left(\frac{z_{1s,L}}{z_{1u,L}}\right)$$

Vertical economy

Setup: Each HH supplies one labor type o to sector i. The most upstream sector in the supply chain sells its output to the second sector, which in turn sells its output to the third and so forth. The last sector sells to FD.



- Both substitution and scale effects

$$
\begin{aligned}
d\ln\left(\frac{w_2}{w_3}\right) &= d\ln\left(\frac{\vartheta_1}{\vartheta_2}\right) + d\ln\left(\frac{\Omega_{1,L}}{\Omega_{2,L}}\right) \\
&= d\ln\Omega_{32} + d\ln\left(\frac{\Omega_{2,L}}{\Omega_{3,L}}\right) \\
&= \left(\frac{\sigma-1}{\sigma}\right) d\ln\left(\frac{z_{2,L}}{z_{3,L}}\right)
\end{aligned}
$$

- Can be generalized for $\sigma_1 \neq \sigma_2$etc.

Conclusion

- **Labor mobility frictions hamper aggregate growth**
  - Due to misallocation of workers across occupations and sectors.
  - Theil index as exact inequality measure from economic principles.
- **Frictions can affect growth and inequality through various channels**
  - growth accounting with frictions.
  - GE framework of output growth and inequality.
  - Productivity and mobility shocks.
  - General non-parametric results and parametric examples.
- **Next**
  - Quantification of channels on US data.