

# Working Paper Research

20 & 21 October 2022 N° 417

NBB conference 2022  
Household Heterogeneity and Policy Relevance

Income inequality in general equilibrium  
by Bastien Bernon, Joep Konings and Glenn Magerman



**Editor**

Pierre Wunsch, Governor of the National Bank of Belgium

**Editorial**

On October 20-21, 2022 the National Bank of Belgium hosted a Conference on *“Household Heterogeneity and Policy Relevance”*.

Papers presented at this conference are made available to a broader audience in the NBB Working Paper Series ([www.nbb.be](http://www.nbb.be)). This version is preliminary and can be revised.

**Statement of purpose:**

The purpose of these working papers is to promote the circulation of research results (Research Series) and analytical studies (Documents Series) made within the National Bank of Belgium or presented by external economists in seminars, conferences and conventions organized by the Bank. The aim is therefore to provide a platform for discussion. The opinions expressed are strictly those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

**Orders**

The Working Papers are available on the website of the Bank: <http://www.nbb.be>.

© National Bank of Belgium, Brussels

All rights reserved.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

ISSN: 1375-680X (print)

ISSN: 1784-2476 (online)

# Income inequality in general equilibrium \*

Bastien Bernon  
ECARES (ULB)

Joep Konings  
Nazarbayev University GSB  
KU Leuven  
and CEPR

Glenn Magerman  
ECARES (ULB)  
and CEPR

September 25, 2022

## Abstract

We develop a quantitative framework in which income inequality arises endogenously in response to productivity shocks. The framework accommodates sectoral input-output linkages, arbitrary elasticities of factors and intermediates, and heterogeneous workers that endogenously choose to supply their labor across sectors. Workers are imperfectly mobile across sectors, parameterized by a Roy-Frechet setup. We characterize the impact of Harrod-neutral shocks and changes in labor mobility on income inequality and welfare up to first- and second order. Inequality arises in equilibrium due to a combination of changes in income share and labor use across all sectors due to their dependencies in the input-output network. We calibrate the model using Belgian data and provide quantitative results, confirming strong non-linearities. These results suggest that labor market-improving policies can have strong effects on both welfare and inequality, but the impact is both quantitatively and qualitatively dependent on the structure of the economy and its initial equilibrium.

**Keywords:** Income inequality, production networks, disaggregated macro models, wage gaps, mobility of workers, heterogeneous factors of production.

**JEL Codes:** D24, D33, D50, D57, D63, E24, J31.

---

\*We would like to thank the participants at the National Bank of Belgium Colloquium 2022 on Household Heterogeneity and Policy Relevance for various comments. Bastien Bernon: [bastien.bernon@ulb.be](mailto:bastien.bernon@ulb.be). Glenn Magerman: [glenn.magerman@ulb.be](mailto:glenn.magerman@ulb.be). Joep Konings: [joep.konings@nu.edu.kz](mailto:joep.konings@nu.edu.kz) and [joep.konings@kuleuven.be](mailto:joep.konings@kuleuven.be).

# 1 Introduction

We develop a quantitative framework that generates income inequality in general equilibrium. In this setup, a competitive economy accommodates sectoral input-output linkages, arbitrary elasticities of factors and intermediates, and heterogeneous workers that endogenously choose to supply their labor across sectors.

Producers combine inputs with heterogeneous labor to produce outputs to be sold again to other producers and final consumers. Workers choose to work in a given sector, determined by their preferences for a sector, the wage in that sector, the elasticity of imperfect mobility of labor across sectors and relative to wages and preferences in all other sectors. Workers are also consumers, linking productivity shocks to income shocks throughout the input-output structure of the economy.

The elasticity of labor mobility is parameterized through a simple Roy-Frechet setup as in [Lagakos and Waugh, 2013] and [Galle and Lorentzen, 2021]. This setup allows for arbitrary labor mobility across sectors, and nests perfect immobility (specific factors models) and perfect mobility (typical for many classical general equilibrium models) as special cases. In partial equilibrium, a change in labor mobility affects the reallocation of workers across sectors. In general equilibrium, wages also adjust, generating potential income inequalities across sectoral wages. Intuitively, if workers are perfectly mobile across sectors, wage inequality is moot in response to a productivity shock: workers reallocate endogenously across sectors such that a single economy-level wage arises in equilibrium. Conversely, in the case of perfect immobility, workers take the full effect of productivity shocks through equilibrium adjustment in wages. More generally, imperfect labor mobility generates income inequality in response to shocks, attenuated by the amount of labor mobility.

We then provide comparative statics for the change in wages in one sector, resulting from a Harrod-neutral (labor-augmenting) productivity shock in any sector. These shocks affect the efficiency units of labor in that sector, and thus change the optimal allocation of labor in all sectors in equilibrium. The total effect of a productivity shock in a given sector  $s$  on wages in any sector  $i$  is given by a change in the income share of households supplying their labor to  $i$  (the labor centrality channel), the change in labor use in  $i$  (the labor supply channel), and a GDP shifter in response to the productivity shock. The first two channels generate inequality, while the GDP shifter only affects welfare levels.

To build intuition, we first document an income-inequality neutral result. In a benchmark Cobb-Douglas production and consumption economy, there is only an effect of the GDP shifter, independent of the amount of labor mobility: while real wages change in accordance to the shift in GDP, all wages move in tandem, resulting in no change in income inequality in response to a shock. The intuition is that labor use and wages offset each other perfectly to keep labor shares in production constant.

In general however, productivity shocks are not inequality-neutral, and are governed by the sign and size of the other two channels. The labor centrality channel dictates the change in the importance of labor supplied in sector  $i$  for final demand (and thus GDP). In particular, it is given by a substitution and scale effect, weighted by the importance of that sector's income in aggregate value added. The substitution effect captures how sectoral labor use adjusts in response to a productivity shock, as prices for intermediates

and labor in all sectors react to the shock. The scale effects captures the change in output in that sector in response to the changes in labor compensation in sector  $i$ . The sector's income share conveys the intuition that low income share sectors are impacted more intensely by both substitution and scale effects, as changes in labor demand and market size are distributed among fewer workers.

Finally, the labor supply channel dictates the reallocation of labor supply across sectors in response to the shock. This reallocation depends on the labor mobility and the change in wage in sector  $i$ , relative to the preference-weighted change in wages in all sectors. Clearly, if labor mobility is low, productivity shocks trigger stark changes in income inequality, as workers cannot move and need to take the wage shock as a result of the productivity shock as given. Conversely, high labor mobility dampens the effect of wage inequality as people move more freely across sectors in response to the shock.

To further highlight the various channels, we provide results for some simple economies that differ in their input-output structure with perfect labor immobility. In a horizontal economy, households provide labor to one of  $\mathcal{N}$  sectors which is the single input of production, and all sectors sell to final demand. In this structure, there is only a scale effect, and no substitution effect: the relative change in wages in two sectors is only determined by the final demand elasticity, and how consumers reallocate their expenses across sectors in response to the productivity shock. In another example, households supply one type of heterogeneous labor to a single sector in the economy. In this roundabout economy, used in classic models of wage gaps (e.g. [Autor et al., 2003] and [Acemoglu and Autor, 2011]), one sector produces everything in the economy and uses all types of heterogeneous labor and its own output in production. In this special case, there is only a substitution effect, but no scale effect. The scale effect is moot as this one sector generates all output in the economy. Wage inequalities appear because of a reallocation of the use of heterogeneous labor in input use. Finally, in a vertical economy, firms only sell to downstream producers up to final demand, combining labor with upstream inputs. In this case, both substitution and scale effects shape income inequality. We then turn to a more general case in which labor is imperfectly mobile. Intuitively, all results on substitution and scale hold, with a dampening effect on income inequality as labor mobility increases.

Next, we fully characterize the impact of productivity shocks on aggregate output and inequality. We generate results for both first-order (linear) and second-order (non-linear) change in output in response to shocks. We provide results for both productivity shocks, as well as a change in labor mobility. In response to a productivity shock, the change in real GDP (and welfare) is given by the labor income shares of each sector. Intuitively, from an envelope theorem argument, income shares do not change in response to the productivity shock, and all effects are linear in the productivity shocks. More generally however, income shares respond endogenously to productivity shocks, and non-linearities appear. This result extends [Baqaee and Farhi, 2019] to account for imperfect factor mobility, which now include the substitution and scale effects discussed above.

As another exercise, we evaluate a change in labor mobility (e.g. due to labor market and educational policies to reduce labor frictions). In this case, up to first order, a change in mobility affects welfare, expressed as the income share weighted sum of the elasticity of labor use with respect to the mobility parameter. If wages tend to be relatively higher in sectors that are more important for final demand through the production network, an

increase in mobility implies an increase in labor supply in these sectors. This generates a positive elasticity of mobility with respect to welfare up to first order. Conversely, if wages are not positively correlated with final demand shares, this elasticity is negative. More generally, wages adjust in equilibrium, in response to a change in mobility. This generates second order effects, in which a change in worker mobility not only generates propagation and aggregate effects through changes in income shares, but also directly through a change in relative wages. The second-order effects recover the scale and substitution effects of a change in labor mobility on aggregate output.

Finally, we calibrate the model using data for 64 sectors in Belgium, and provide quantitative first-order and second-order results on both welfare and wage inequality. We simulate the effect of sector-specific labor productivity shocks, and changes in labor mobility. We first simulate the effect of a labor-specific productivity shock on aggregate output and income inequality. For a given parameterization of elasticities of substitution (strategic complements) and a labor mobility close to perfect mobility, a 1% labor productivity shock in the energy sector corresponds to a 0.005% change in aggregate welfare, up to a first order. Taking into account the equilibrium adjustments of income shares with imperfect mobility shows the non-linearities of the impact on welfare, with an elasticity of -0.007. Turning to the effect on wage inequality, and for the same calibration, we find that wages across sectors on average respond with an elasticity of 0.004. However, there is dispersion in the wage responses across sectors, with a standard deviation of 0.18. It turns out that the vast majority of the wage changes is driven by the labor centrality channel, partly offset by the labor supply channel. When evaluating the impact of a change of labor mobility, we also provide results for welfare and income inequality. Up to first order, a 1% increase in labor mobility generates a 0.14% increase in welfare. The second-order effects however show strong concavity, suggesting very different policy implications, depending on what the level of mobility is before the shock. For low levels of mobility, providing slack on mobility has strong positive effects on welfare. Turning to the impact on wage inequality, we find strong and divergent effects across sectors.

## 2 An Economy with input-output linkages and imperfect labor mobility

### 2.1 Model description

We start by formulating an economy with sector input-output linkages, arbitrary elasticities of factors and intermediates, and heterogeneous workers that supply their labor across sectors.

**Production** The production side consists of  $\mathcal{N}$  sectors. Representative firms in each of these sectors produce a homogeneous good, using labor and goods from other sectors to produce their output following a constant returns-to-scale technology.

$$y_i = F_i(z_{il}, l_i, \{x_{ij}\}_{j \in \mathcal{N}})$$

where  $y_i$  is sectoral output,  $F_i$  is the production function that transforms inputs into outputs,  $z_{il}$  is a Harrod-neutral productivity shock specific to labor in sector  $i$ ,  $l_i$  is the labor use in sector  $i$  and  $x_{ij}$  is the use of the good produced in sector  $j$  in sector  $i$ .

Production takes place in perfect competition. Hence, sector prices are equal to marginal costs, so that:

$$p_i = C_i(z_{il}, w_i, \{p_j\}_{j \in \mathcal{N}})$$

where  $p_i$  is the price of the good in sector  $i$ ,  $C_i$  is the cost function,  $w_i$  is the wage of workers in sector  $i$ , and  $p_j$  is the price of input  $j$ .

**Workers** There is a measure one of workers who supply labor. The sector in which they supply their labor depends on their preferences and the wage that they would earn in each sector. Preferences follow a joint Fréchet distribution, with parameter  $\Phi_i$ , which is a location parameter that can vary across sectors  $i \in \mathcal{N}$ , and a dispersion parameter  $\kappa = \frac{\theta}{1-\theta}$  which governs the mobility of workers between sectors. The share of workers supplying their labor to sector  $i$  is equal to :

$$l_i = \frac{\Phi_i w_i^{\frac{\theta}{1-\theta}}}{\mathcal{W}^{\frac{\theta}{1-\theta}}}$$

where  $w_i$  is the wage in sector  $i$  and  $\mathcal{W}$  is the wage index of workers equal to  $\left(\sum_j \Phi_j w_j^{\frac{\theta}{1-\theta}}\right)^{\frac{1-\theta}{\theta}}$ .

When  $\theta \rightarrow 0$ , workers are immobile between sectors, so that:

$$\lim_{\theta \rightarrow 0} l_i = \frac{\Phi_i}{\sum_j \Phi_j}$$

which depends only on the preferences of workers regarding the sector in which they want to work. Then labor supply does not depend on wages.

Conversely, when  $\theta \rightarrow 1$ , workers are perfectly mobile between sectors, and we have that:

$$\begin{cases} \lim_{\theta \rightarrow 1} l_i = 0 & \text{if } \exists w_i < w_j \text{ with } j \in \mathcal{N} \\ \lim_{\theta \rightarrow 1} l_i = +\infty & \text{if } w_i > w_j \forall j \in \mathcal{N} \\ \lim_{\theta \rightarrow 1} l_i = \frac{\Phi_i}{\sum_j \Phi_j} & \text{if } w_i = w_j \forall j \in \mathcal{N} \end{cases}$$

which implies that workers would all move to the same sector where the wage is the highest. This is a partial equilibrium analysis where wages are exogeneous. Once they are endogeneous to the model, the labor supply would converge to the third case where all wages are equal which is the classical result for perfect workers' mobility.

**Consumption** Workers are also consumers, sharing identical homothetic preferences:

$$\mathcal{Y} = D(\{c_i\}_i)$$

where  $\mathcal{Y}$  is the real output of this economy,  $D$  is an aggregator of consumption equal to welfare and real GDP in this economy and  $c_i$  is the final demand for the good from sector  $i$ . Total income of the group of workers supplying labor to sector  $i$  is equal to  $w_i l_i$ , which pins down the budget constraint of each household.

**Equilibrium** Finally, market-clearing conditions imply that all labor and goods markets clear:

$$L = 1 = \sum_i l_i; \quad y_i = c_i + \sum_j x_{ji}$$

**Input-output definitions** In this economy, the technical coefficient matrix  $\Omega$  is defined as a  $\mathcal{N} \times \mathcal{N}$  matrix whose  $ij$ -th element is :

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}$$

The technical coefficients represent the cost share of the use of intermediate good  $j$  in total output of  $i$ .

The  $\mathcal{N} \times \mathcal{N}$  Leontief matrix  $\Psi$  is built on the technical coefficient matrix :

$$\Psi = (I - \Omega)^{-1}$$

where  $I$  is the identity matrix. The  $ij$ -th element  $\Psi_{ij}$  of this matrix denotes the direct and indirect use of sector  $j$  as a supplier to sector  $i$ .

We define the labor matrix  $\Omega_L$  as a vector of dimension  $1 \times \mathcal{N}$ , whose  $i$ -th element is the labor compensation allocated to workers in sector  $i$ :

$$\Omega_{il} = \frac{w_i l_i}{p_i y_i}$$

Similarly, we define the vectors of final consumption  $\Omega_C$ , whose  $i$ -th element is:

$$\Omega_{ci} = \frac{p_i c_i}{GDP}$$

which gives the final demand for good  $i$  as a share of GDP.

These variables allow to define the importance of sectors and groups of households for the production of real GDP,  $\mathcal{Y}$ . By accounting definition, real GDP is equal to the final demand homothetic function  $D$ . Therefore the importance of sectors and households for production is given by their direct and indirect importance in production of final demand goods. The sector sales shares of nominal GDP,  $\lambda_i = \frac{p_i y_i}{GDP}$  which are the Domar weights, and the income shares of household groups,  $\Lambda_i = \frac{w_i l_i}{GDP}$ , embody this intuition as follows :

$$\Lambda' = \Omega_C' \Psi \text{diag}(\Omega_L) ; \quad \lambda' = \Omega_C' \Psi$$



where the  $i$ -th element of vector  $\lambda$  and  $\Lambda$  gives the direct and indirect use of sector  $i$  and labor supplied to sector  $i$  for the production of final goods :

$$\Lambda_i = \sum_j \Omega_{cj} \Psi_{ji} \Omega_{il} ; \quad \lambda_i = \sum_j \Omega_{cj} \Psi_{ji}$$

Hence, the importance of a household for the real output  $\mathcal{Y}$  is given by the direct and indirect importance of the labor it supplies to the production of final demand goods. The same intuition follows for sectors. This importance is given respectively by their income shares and their sales shares of nominal GDP.

## 2.2 Comparative statics

We now analyze comparative statics of the network economy developed above, starting from the initial equilibrium where the labor-specific productivity variables  $z_{sl}$ , are set to 1. Hence, taking the log-changes of income shares  $w_i l_i / GDP$  with respect to a Harrod-neutral shock in sector  $s$  and isolating the log-change in wage, we have that:

$$\frac{d \log w_i}{d \log z_{sl}} = \underbrace{\frac{d \log \Lambda_i}{d \log z_{sl}}}_{\text{labor centrality channel}} - \underbrace{\frac{d \log l_i}{d \log z_{sl}}}_{\text{labor supply channel}} + \underbrace{\frac{d \log GDP}{d \log z_{sl}}}_{\text{Aggregate channel}} \quad \forall i \in \mathcal{N}$$

This equation is central to the understanding of income inequality in network economies and provides the basic intuition for the rest of the paper.

First, the *labor centrality channel* represents the change in the importance of labor supplied in sector  $i$  for final demand and therefore for GDP. It is given by:

$$\frac{d \log \Lambda_i}{d \log z_{sl}} = \underbrace{\frac{1}{\Lambda_i}}_{\text{inertia multiplier}} \left( \underbrace{d \Omega_{il} \lambda_i}_{\text{substitution effect}} + \underbrace{\Omega_{il} d \lambda_i}_{\text{scale effect}} \right)$$

where the change in income shares  $\Lambda_i$  appear through a change in labor demand  $d \Omega_{il}$ , holding sales shares fixed, and/or through a change in market size  $d \lambda_i$ , holding labor supply fixed. These effects are respectively the substitution and scale effects. When a productivity shock occurs in one or more sectors of the economy, intermediate goods and labor have new equilibrium prices and wages that clear the markets. Firms and consumers update their optimal decision and new labor shares and sector sizes result in equilibrium. These effects are adjusted by  $\frac{1}{\Lambda_i}$  which we call an inertia multiplier. This multiplier conveys the intuition that income shares reflecting a low level of labor compensation with respect to GDP would be impacted more intensely as the effects of changes in labor demand and market size would be distributed among fewer workers.

Second, the *labor supply channel* indicates the impact of workers' reallocation between sectors on labor supply in one sector. This channel is given by :

$$\frac{d \log l_i}{d \log z_{sl}} = \left( \frac{1 - \theta}{\theta} \right) \Phi_i \left( d \log w_i - \sum_j \Phi_j d \log w_j \right)$$

Preferences of workers are fixed, but wages in each sector change in response to the labor-sector productivity shock, creating an incentive for workers to move. When  $\theta$  is close to zero, workers will not move, even if wages differ a lot between sectors. Hence, productivity shocks will trigger stark changes in wage inequality, as workers will not be able to respond to the shock by moving across sectors. This coincides with a specific factors model. In the other limit, when  $\theta$  is close to 1, workers will immediately adapt to small changes in wages and therefore wage inequality would not be able to take shape. This is the standard case in which wages clear in general equilibrium with perfect mobility. Finally, the *aggregate channel* specifies the importance of the change in GDP for the sectoral wage. We have that :

$$\frac{d \log GDP}{d \log z_{sl}} = \Lambda_s$$

which is an implication of Hulten's theorem for factors of production. This channel produces a scale effect on wage inequality as it impacts all sectoral wages in the same way. We have that if the two first channels are muted and the shock only impacts wages through the aggregate channel then there is no change in wage inequality. We elaborate on this intuition in the next section to develop an *inequality-neutrality result* which is central for the understanding of income inequality in production networks.

### 3 An Inequality-Neutral Result

To understand how changes in income inequality occur in network economies, we start with a benchmark framework where production functions and household preferences are Cobb-Douglas and workers are immobile ( $\theta = 0$ ):

$$y_i = l_i^{\omega_{il}} \Pi_j x_{ij}^{\omega_{ij}} \quad ; \quad U = \Pi_j c_i^{\omega_{ci}} \quad ; \quad l_i = \frac{\Phi_i}{\sum_j \Phi_j}$$

By the first-order conditions, we have that the shares of expenses for intermediate good and labor of the firms with respect to their total sales and the share of expenses by good of households are fixed. Hence in Cobb-Douglas economies we have that :

$$\omega_{il} = \Omega_{il} \quad \forall i \in \mathcal{N} \quad ; \quad \omega_{ij} = \Omega_{ij} \quad \forall i, j \in \mathcal{N} \quad ; \quad \omega_{ci} = \Omega_{ci} \quad \forall i \in \mathcal{N}$$

which means that the change in expenses and in total sales (or in total incomes for households) always compensate in order to keep a share of expenses fixed in this simple model. In this case where the changes in intermediate good, factor shares and consumption shares are always equal to zero and knowing that the change in Domar weights  $\lambda_i$  only depend on the change in these shares, we also have that:

$$d \lambda_i = \sum_o \Psi_{oi} \left( \underbrace{d \Omega_{co}}_{=0} + \sum_j \underbrace{d \Omega_{jo}}_{=0} \lambda_j \right) = 0 \quad \forall i \in \mathcal{N} \text{ in this model}$$

which conveys the intuition that the change in the importance of a sector  $i$  compared to GDP is given by the change in final sales and intermediate good sales of the other sectors  $j \in \mathcal{N}$  weighted by the importance of sector  $i$  as a supplier to these other sectors. Therefore, building on the first-order conditions, we have that  $d\lambda_i = 0 \forall i \in \mathcal{N}$  in Cobb-Douglas economies which implies that the relative size of each sector with respect to GDP does not change.

These results imply that the *labor centrality channel* is silent in this model. Then, following the assumption that workers are immobile and therefore cannot change sectors, we have that the *labor supply channel* is also silent in this model. Both these results imply that the only channel impacting wages in this model is the *aggregate channel* which is the same for each sector's wage. Hence, we have that each worker is impacted in the same way, and therefore any labor-specific productivity shock will only make each worker richer or poorer in real terms, but will not impact wage inequality. We formalize this intuition in theorem 1 for any model implying inequality-neutral results.

**Theorem 1 (Inequality-neutrality)** *Assume a Harrod-neutral productivity shock specific to labor occurs in the economy. Then the two following propositions are correct if and only if the third is:*

$$(1) \frac{d \log \Lambda_i}{d \log z_{jl}} = 0 \quad \forall i \in \mathcal{N}$$

$$(2) \frac{d \log l_i}{d \log z_{jl}} = 0 \quad \forall i \in \mathcal{N}$$

$$(3) \frac{d \log w_i}{d \log z_{jl}} = \frac{d \log GDP}{d \log z_{jl}} \quad \forall i \in \mathcal{N} \quad \text{and} \quad d \log \left( \frac{w_i}{w_j} \right) = 0 \quad \forall i, j \in \mathcal{N}$$

**Corollary 1** *A corollary of theorem 1 is its negative statement. At least one of the two following propositions is correct if and only if the third is:*

$$(1) \exists i \in \mathcal{N} : \frac{d \log \Lambda_i}{d \log z_{jl}} \neq 0$$

$$(2) \exists i \in \mathcal{N} : \frac{d \log l_i}{d \log z_{jl}} \neq 0$$

$$(3) \exists i, j \in \mathcal{N} : d \log \left( \frac{w_i}{w_j} \right) \neq 0$$

In the next section, we build on this benchmark result to study separately the channels creating wage inequality in equilibrium using simple network economies.

## 4 Simple structures

In this section, we study simple economic structures to specify the different channels through which changes occur in wage inequality, following labor-specific productivity

shocks. To do so, we study the case when firms use a sector-specific CES technology and households have CES preferences:

$$y_i = \left( \omega_{il} \frac{1}{\sigma_{y,i}} l_i^{\frac{\sigma_{y,i}-1}{\sigma_{y,i}}} + \sum_j \omega_{ij} \frac{1}{\sigma_{y,i}} x_{ij}^{\frac{\sigma_{y,i}-1}{\sigma_{y,i}}} \right)^{\frac{\sigma_{y,i}}{\sigma_{y,i}-1}} ; \quad U = \left( \sum_j \omega_{cj} \frac{1}{\sigma_c} c_j^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c-1}}$$

where  $\sigma_{y,i}$  is the elasticity of substitution between inputs in the production process of sector  $i$ ,  $\sigma_c$  is the elasticity of substitution between consumption goods for households and the  $\omega$  parameters are the shares of inputs and consumption goods at the initial equilibrium. In this setup, theorem 1 no longer holds, and productivity shocks can generate labor inequality effects.

We proceed to the study of different stylized economies and study what basic production structure can teach us about changes in wage inequality when **1)** labor is specific to each sector and when **2)** labor is imperfectly mobile.

We start with the case where labor is specific to a sector ( $\theta \rightarrow 0$ ). Each of the following economies represent a different combination of channels impacting wage inequality. The first, the horizontal economy, only opens the change in market channel but keeps the labor demand channel closed. Second, in the roundabout economy, only the labor demand channel is opened and the market size do not change. Finally, in the vertical economy, both channels are opened. We study what is the sign and size of the effects implied by these channels for wage inequality.

**Horizontal economy** See [Figure 1](#). Households supply one type of labor specific to one sector, while sectors only use labor to produce the output that they sell directly to final consumers:

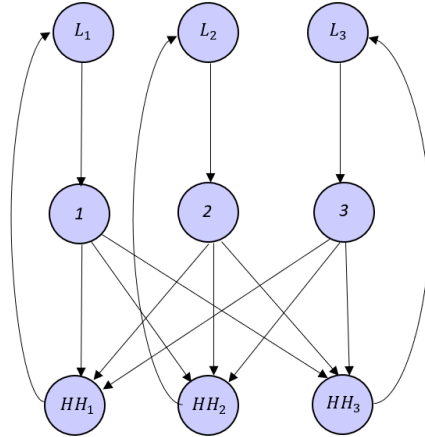


Figure 1: Horizontal economy

In this economy, firms only sell to final demand and their sales shares only depend on final consumers. As sectors only use labor, sales shares and income shares coincide ( $\lambda_i = \Lambda_i$ ) and household supplying labor in sector  $i$  is the only one impacted by the change in

sales share of sector  $i$ . Hence, the sign of the effect on wage inequality depends on final demand's elasticity of substitution between consumption goods and how final consumers reallocate their expenses between one sector and another. The change in sales shares is the only effect impacting wage inequality, as labor is the only input and cannot be substituted by another input. This implies that there are only scale effects but no substitution effect:

$$\begin{aligned} d \log \left( \frac{w_1}{w_2} \right) &= d\Omega_{c1} - d\Omega_{c2} \\ &= (1 - \sigma_c) \left( \Omega_{c1} d \log \left( \frac{w_1}{z_{1l}} \right) - \Omega_{c2} d \log \left( \frac{w_2}{z_{2l}} \right) - (\Omega_{c1} - \Omega_{c2}) d \log \mathcal{P} \right) \end{aligned}$$

where  $\mathcal{P}$  is the consumers' price index in this model. The sign of the wage inequality depends on  $\sigma_c$ . If it is bigger than 1, consumption goods are imperfectly substitutable. A positive productivity shock incurs a positive effect on wages as the demand for the good produced with that labor would increase. Hence, as firms compete for final demand, workers also ultimately compete across sectors and more productive specialized workers earn a higher wage. On the other hand, if  $\sigma_c$  is smaller than 1, consumption goods are complements. Workers that become more productive are penalized by a decrease in demand for the good that they produce. Workers are better off when their own productivity decreases as households have to reallocate more resources to that sector and the real wage mechanically increases.

The size of the change in wage inequality depends on the intensity with which households reallocate their demand from one good to another and therefore favor some specialized workers over others. This depends on the difference between  $\sigma_c$  and 1, on the size of the shock  $d \log z_{sl}$  and on the parameters  $\Omega_C$ .

**Roundabout economy** In this example, each household supplies one type of labor, see [Figure 2](#). All labor is used by one sector, which subsequently sells its output to itself and to final consumers. One sector produces everything in the economy and uses two sorts of labor and its own output in production.<sup>1</sup> In this setup, only substitution matters as sector 1 will always produce the entirety of real output in this economy. Therefore, when a labor-specific productivity shock occurs, wage inequality changes as follows:

$$\begin{aligned} d \log \left( \frac{w_{1,A}}{w_{1,B}} \right) &= d\Omega_{1l,A} - d\Omega_{1l,B} \\ &= (1 - \sigma_{y,1}) \left( \Omega_{1l,A} d \log \left( \frac{w_{1,A}}{z_{1l,A}} \right) - \Omega_{1l,B} d \log \left( \frac{w_{1,B}}{z_{1l,B}} \right) \right) \end{aligned}$$

where the sign of the change in wage inequality is dictated by  $\sigma_{y,1}$ , i.e. the elasticity of substitution between inputs in sector 1. The size of the shock is determined by the difference between the elasticity and 1, by the size of the shock and now by the parameters  $\Omega_L$ .

---

<sup>1</sup>This economy is almost identical to the one developed in the classical model of study of wage gaps (see e.g. [Autor et al., 2003](#)) and [Acemoglu and Autor, 2011](#)).

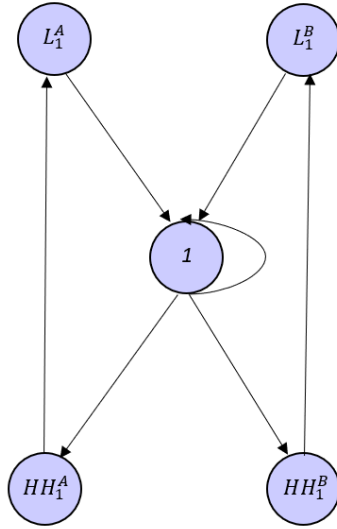


Figure 2: Roundabout economy

**Vertical economy** Each household supplies its labor to a different sector at a different position in the supply chain, see Figure 3. The most upstream sector in the supply chain sells its output to the second sector, which in turn sells its output to the third and so forth. Each sector, except the last one, use labor and the output from the upstream sector as input. Firms in the last sector only use the output from its upstream sector as input and sell their output to the most downstream final consumers.

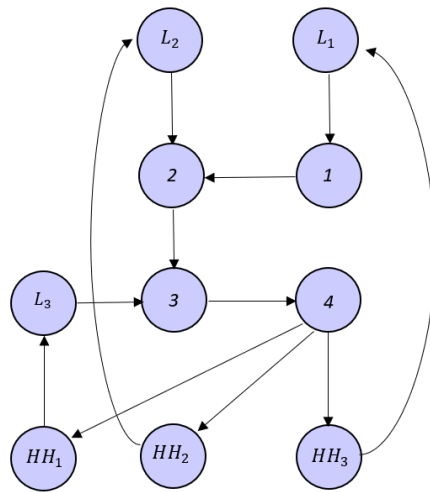


Figure 3: Vertical economy

The intuition developed in the horizontal and roundabout economies applies jointly as there are now substitution and scale effects when a shock occurs. The change in wage inequality follows:

$$d \log \left( \frac{w_1}{w_2} \right) = \lambda_2 \left( \frac{d\Omega_{21}}{\Lambda_1} - \frac{d\Omega_{2l}}{\Lambda_2} \right) + d\lambda_2 \left( \frac{\Omega_{21}}{\Lambda_1} - \frac{\Omega_{2l}}{\Lambda_2} \right)$$

and the change in wage inequality depends on the elasticity of substitution between inputs in the own sector and in the downstream sector. The labor share depends on the labor demand in the own sector and the sales share depends on the input demand in the downstream sector. We have that :

$$\Omega_{21} = 1 - \Omega_{2l} \quad ; \quad d\Omega_{21} = 1 - d\Omega_{2l} \quad ; \quad d\lambda_2 = d\Omega_{32}\lambda_3$$

Hence, we have that the change in wage inequality in the vertical economy jointly depends on the difference between  $\sigma_{y,2}$  and  $\sigma_{y,3}$  and 1, the size of the shock and the parameters  $\Omega_L$ .

**Imperfectly mobile labor** We now turn to an economic structure when  $\theta$  does not tend to zero and therefore labor is (im)perfectly mobile between sectors in [Figure 4](#). The households supply their labor to each sector with preferences to work in one sector or another. Their labor supply is readjusted as wages adapt and dampen the impact of the shock. Firms in each sector only use labor as input and sell their output directly to final consumers.

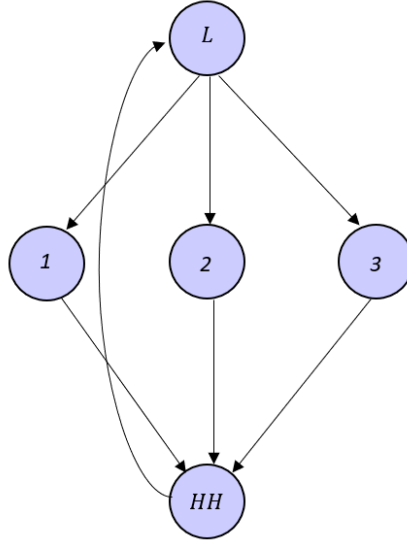


Figure 4: Allocation economy

This structure can be compared to the horizontal economy where  $\theta$  does not tend to zero and where therefore the labor channel of wage inequality has been unlocked. Hence, wage inequality follows:

$$\begin{aligned} d \log \left( \frac{w_1}{w_2} \right) &= d\Omega_{c1} - d\Omega_{c2} - \left( \frac{\theta}{1-\theta} \right) d \log \left( \frac{w_1}{w_2} \right) \\ &= (1-\theta) (d\Omega_{c1} - d\Omega_{c2}) \end{aligned}$$

where the case of the horizontal economy is included when  $\theta = 0$  and where the change in wage inequality is equal to zero when labor is perfectly mobile ( $\theta = 1$ ). The labor supply channel works as a dampening effect applied to the channels already developed above. The more mobile are workers, the more mitigated will be the change in wage inequality.

## 5 General characterization

We develop the general characterization in this economy that we use in our calibrated exercises. We seek for two results in network economies, the real GDP impact of the shock and the dispersion change in wages. To do so, we develop aggregate and dispersion characterization for productivity shocks and exogeneous changes in workers' mobility. For the productivity shocks, we extend the results in [Baqae and Farhi, 2019] by introducing imperfect workers' mobility and by using factor-specific shocks. For the change in workers' mobility  $\kappa$ , we develop new results to the literature. We start with the aggregate and dispersion results for productivity shocks.

### 5.1 Labor-specific productivity shock

The aggregate impact of a sector-labor specific productivity shock on real GDP  $\mathcal{Y}$  is up to a second-order approximation, given by the first-order and second-order effect of the shock. By Hulten's theorem using the envelope theorem, the first-order elasticity of real GDP to the labor-sector productivity shock is the income share of the households supplying labor in the shocked sector:

$$\frac{d \log \mathcal{Y}}{d \log z_{sl}} = \Lambda_s$$

This first-order impact is also the final result we would find in Cobb-Douglas network economies as explained in section 3, because these income shares would not change and the effect would be fully linear. Nevertheless, outside of this specific framework, income shares are endogenous and second-order effects appear:

$$\frac{d^2 \log \mathcal{Y}}{d \log z_{sl}^2} = \frac{d \Lambda_s}{d \log z_{sl}}$$

This is the main result in [Baqae and Farhi, 2019] applied to a labor-sector specific productivity shock. However, our results are different because workers' imperfect mobility creates endogeneous labor supply. When we take the full derivation of the income shares  $\Lambda_i$ , we find that:

$$\frac{d \Lambda_i / \Lambda_i}{d \log z_{sl}} = \frac{d \log \Lambda_i}{d \log z_{sl}} = \frac{d \log w_i}{d \log z_{sl}} + \frac{d \log l_i}{d \log z_{sl}} - \frac{d \log GDP}{d \log z_{sl}}$$

where we observe that the change in income shares  $d \log \Lambda_i / d \log z_{sl}$  depends on the share in labor supply  $d \log l_i / d \log z_{sl}$ . Now, deriving the second-order approximation of  $d \log \mathcal{Y}$ ,



we have that:

$$\begin{aligned} d \log \mathcal{Y} &= \frac{d \log \mathcal{Y}}{d \log z_{sl}} d \log z_{sl} + \frac{1}{2} \frac{d^2 \log \mathcal{Y}}{d \log z_{sl}^2} (d \log z_{sl})^2 \\ &= \underbrace{\Lambda_s d \log z_{sl}}_{\text{1st order effect}} + \underbrace{\frac{1}{2} \frac{d \Lambda_s}{d \log z_{sl}} (d \log z_{sl})^2}_{\text{2nd order effect}} \end{aligned}$$

Hence, once we find the change in wages or income shares in our economy, we are able to find the change in real GDP  $\mathcal{Y}$  and the dispersion of wage elasticities  $d \log w_i / d \log z_{sl}$ . To find the system of equations solving the wage elasticities with respect to productivity shocks, start with the following equation for the change in income share of the  $i$ -th group of households:

$$\frac{d \log \Lambda_i}{d \log z_{sl}} = \underbrace{\hat{\sigma}_{y,i} d \log \left( \frac{w_i}{p_i} \right)}_{\text{substitution effect}} + \underbrace{\frac{1}{\lambda_i} \hat{\sigma}_c \sum_o \Omega_{co} \Psi_{oi} d \log \left( \frac{p_o}{\mathcal{P}} \right) + \sum_k \frac{\lambda_k}{\lambda_i} \hat{\sigma}_{y,k} \sum_o \Omega_{ko} \Psi_{oi} d \log \left( \frac{p_o}{p_k} \right)}_{\text{scale effect}}$$

This is the full-blown equation presented in section 3 for  $d \log \Lambda_i / d \log z_{sl}$  where  $\hat{\sigma}_y = (1 - \sigma_y)$ .

The sign of the change in income shares depends on the elasticities of substitution in each sector  $\sigma_{y,k}$  and in final demand  $\sigma_c$  and on the comparative price change of sector  $i$  and the labor used in  $i$  compared to all the other sectors and to all the other labors.

For example, take a positive labor-specific shock in sector  $j$  in an economy where all the elasticities of substitution are strictly smaller than 1. In this economy with strategic complementarities, the more competitive is sector  $i$  following the shock, the more it loses in market share and the more the group of households  $i$  loses in income share. In the first part of the equation, when the equilibrium wage paid to households  $i$  decreases with respect to the cost of all the other inputs of sector  $i$  as reflected by  $p_i$ ,  $\Lambda_i$  is impacted negatively. In the second part of the equation, the importance of sector  $i$  as a supplier kicks in. If the downstream sectors to  $i$  become cheaper, as a result of a smaller price of  $i$  or because they are more exposed to the positive shock, then the importance of  $i$  as a supplier decreases as the expenses of final consumers and downstream sectors are allocated elsewhere.

These consequences of the shock are conditional on the value of the elasticities compared to one as explained in the simple structure economies. When the elasticities are strictly greater than one, the mechanisms are reversed.

Study now the change in labor supply:

$$\frac{d \log l_i}{d \log z_{sl}} = \left( \frac{\theta}{1 - \theta} \right) \frac{\Phi_i}{l_i} \left( \frac{d \log w_i}{d \log z_{sl}} - \frac{d \log \mathcal{W}}{d \log z_{sl}} \right)$$

where  $d \log \mathcal{W} = \sum_o \Phi_o d \log w_o$ .

From the perspective of the households, what matters is the wage earned in the sector in which they are currently working (here sector  $i$ ) compared to all the other sectors weighted by preferences. When the wage in sector  $i$  increases, households from other

sectors move to sector  $i$  conditional on their possibility to move governed by elasticity  $\theta$  and conditional on their vector of preferences  $\Phi$ . When the distance between  $\log w_i$  and  $\log \mathcal{W}$  increases, workers move to sector  $i$  with an elasticity of  $\left(\frac{\theta}{1-\theta}\right) = \kappa$ .

Hence in our setup, the log-changes in wages are:

$$\frac{d \log w_i}{d \log z_{sl}} = \underbrace{\hat{\sigma}_{y,i} d \log \left(\frac{w_i}{p_i}\right) + \frac{1}{\lambda_i} \hat{\sigma}_c \sum_o \Omega_{co} \Psi_{oi} d \log \left(\frac{p_o}{\mathcal{P}}\right) + \sum_k \frac{\lambda_k}{\lambda_i} \hat{\sigma}_{y,k} \sum_o \Omega_{ko} \Psi_{oi} d \log \left(\frac{p_o}{p_k}\right)}_{\text{labor centrality channel}} - \underbrace{\kappa \frac{\Phi_i}{l_i} d \log \left(\frac{w_i}{\mathcal{W}}\right)}_{\text{labor supply channel}} + \underbrace{\Lambda_s}_{\text{aggregate channel}}$$

Noting that  $d \log p_i = \sum_j \Psi_{ij} \Omega_{jl} d \log (w_j / z_{jl})$ , we solve this system of  $\mathcal{N}$  linear equations where the  $\mathcal{N}$  unknowns are the log-changes in wages.

## 5.2 Shock to imperfect workers' mobility

First, the aggregate impact of a shock to the parameter  $\kappa$  controlling imperfect workers' mobility would impact labor supply in each sector. Hence, following an envelope theorem where only direct effect matters, we have that:

$$\frac{d \log \mathcal{Y}}{d \log \kappa} = \sum_j \Lambda_j \frac{d \log l_j}{d \log \kappa}$$

This result follows the same logic as Hulten's theorem where the elasticity of real GDP to labor supply in one sector is given by the income share of workers supplying their labor in that sector  $\Lambda$ . However, as the workers' mobility elasticity  $\kappa$  impacts all sectors, we have the result above where all sectors are impacted.

Second, deriving the elasticity of labor supply to change in workers' mobility elasticity, we find that :

$$\frac{d \log l_j}{d \log \kappa} = \kappa \left( \log w_j - \sum_o l_o \log w_o \right)$$

And hence the first-order impact of a change in  $\kappa$  on real GDP is given by :

$$\frac{d \log \mathcal{Y}}{d \log \kappa} = \kappa \sum_j \Lambda_j \left( \log w_j - \sum_o l_o \log w_o \right)$$

We are interested in dispersion effects and this first-order result does not take into account the endogenous change in wages following the change in  $\kappa$ . So we study the real GDP change following the shock up to a second-order approximation to incorporate the impact

of the change in wages on real GDP and on wage dispersion. Take the following second-order impact equation:

$$\begin{aligned}\frac{d^2 \log \mathcal{Y}}{d \log \kappa^2} &= \frac{d \left( \kappa \sum_j \Lambda_j (\log w_j - \sum_o l_o \log w_o) \right)}{d \log \kappa} \\ &= \frac{d \log \mathcal{Y}}{d \log \kappa} + \sum_j \frac{d \Lambda_j}{d \log \kappa} \left( \log w_j - \sum_o l_o \log w_o \right) + \sum_j \Lambda_j \left( \frac{d \log w_j}{d \log \kappa} - \sum_o l_o \frac{d \log w_o}{d \log \kappa} \right)\end{aligned}$$

here we have that a shock to workers' mobility do not only propagate in the economy through changes in income shares but also directly through the change in comparative wages :  $d \log w_j - d \log \mathcal{W}$ .

Now we are able to find the change in wages following the shock to workers' mobility which allows us to compute its impact on real GDP up to a second-order approximation and its impact on wages dispersion. We have that:

$$\begin{aligned}d \log \mathcal{Y} &= \frac{d \log \mathcal{Y}}{d \log \kappa} d \log \kappa + \frac{1}{2} \frac{d^2 \log \mathcal{Y}}{d \log \kappa^2} (d \log \kappa)^2 \\ d \log \mathcal{Y} &= \underbrace{\left( \kappa \sum_j \Lambda_j \left( \log w_j - \sum_o l_o \log w_o \right) \right)}_{\text{1st order effect}} d \log \kappa + \\ &\underbrace{\frac{1}{2} \left( \frac{d \log \mathcal{Y}}{d \log \kappa} + \kappa \sum_j \frac{d \Lambda_j}{d \log \kappa} \left( \log w_j - \sum_o l_o \log w_o \right) + \kappa \sum_j \Lambda_j \left( \frac{d \log w_j}{d \log \kappa} - \sum_o l_o \frac{d \log w_o}{d \log \kappa} \right) \right)}_{\text{2nd order effect}} (d \log \kappa)^2\end{aligned}$$

where the first-order impact depends on the alignment between income shares and comparative wages and the second-order impact depends on the change in this alignment. Start with the first-order effect, if wages tend to be bigger than other wages in sectors that are more important for final demand, then an increase in workers' mobility implies an increase in labor supply in these sectors that have an higher impact for real GDP. Therefore the elasticity is positive in that case. The reverse is true if wages are not aligned with the importance of sectors for final demand.

Concerning the second-order effects, it recovers the scale effect of a change in  $\kappa$  which is accounted for by the first-order effect and the impact of the changes in incomes shares  $\Lambda$  and wages on the alignment between the importance of sectors for final demand and wages. When this second-order elasticity is positive, it comes exacerbate positive shocks and dampens negative shocks. On the other hand, when this elasticity is negative, it amplifies negative shocks and dampens positive ones. We elaborate on this in the results section.

## 6 Data and Calibration

In this section, we calibrate the model using data on the Belgian economy in order to simulate the impact of productivity and workers' mobility shocks on real GDP and wage dispersion. First, we describe the data that we use and present some descriptive statistics on the Belgian economy. Second, we present the version of the model that we use for this exercise and how we calibrate it.

**Description of the data** We use the dataset "Input-Output Tables 2015 - bis" from the Federal Planning Bureau. This dataset includes the Input-Output tables for the Belgian economy at the disaggregation of 64 industries (NACE REV. 2), the different components of the final demand and of the value-added disaggregated at the sector level. The table can be described as follow:

supply (i)/use (j)	sector 1	...	sector n	...	sector N	FD	output
sector 1	$sales_{11}$	...	$sales_{1n}$	...	$sales_{1N}$	$FD_1$	$output_1$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
sector n	$sales_{n1}$	...	$sales_{nn}$	...	$sales_{nN}$	$FD_n$	$output_n$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
sector N	$sales_{N1}$	...	$sales_{nN}$	...	$sales_{NN}$	$FD_N$	$output_N$
Taxes	$t_1$	...	$t_n$	...	$t_N$	0	0
ROW	$R_1$	...	$R_n$	...	$R_N$	0	0
Labor	$L_1$	...	$L_n$	...	$L_N$	0	0
Capital	$K_1$	...	$K_n$	...	$K_N$	0	0

Where  $sales_{n1}$  corresponds to the sales of sector  $n$  to sector 1,  $FD_1$  is the final demand for goods from sector 1,  $output_1$  is the total output of sector 1,  $ROW$  is the rest of the world,  $R_n$  is the use of intermediate and final goods from the rest of the world for the production of goods in sector  $n$  and  $Taxes$ ,  $Capital$  and  $Labor$  are the other expenses of the using firms in order to produce their goods.

In order to compute the mean income in a sector, we use the data on domestic employment by sector for the fourth quarter of 2015 from the National Bank of Belgium dataset. Then, we take the simple mean annual income in each sector dividing the employee compensation in that sector by the number of workers.

Considering capital incomes in this economy, we sum net operating surplus and consumption of fixed capital in each sector (total gross operating surplus) to find sectoral capital use. We have that the share of capital incomes in this economy equal to 45% of gross value-added while employee compensation represents around 55%.

**Descriptive statistics** In [figure 12](#), we observe that the allocation of workers is very heterogeneous across sectors, where the top five sectors represent 38% of total employment in the Belgian economy. The correspondence table between the sectors' digits and their name is available in the appendix of the present work. In [figure 13](#), we have the average

wages per sector, calculated as sectoral labor compensation divided by sectoral employment. There exists a dispersion between sectoral wages, which varies from 9,000 euros in the "crop and animal production" sector to 153,000 euros in the "manufacture of basic pharmaceutical products" sector.

**Model description** The model that we calibrate is a version of the general framework described above to which we add imports and exports in the IO table. For Belgium, total imports account for 45% as a share of gross value added (GVA) and total exports account for 53% of GVA.

To be consistent with the planning bureau tables, we have imports as a column vector in the IO table, hence only supplying other sectors and not using any input, behaving then as a factor. And we have exports as a row vector, hence only consuming from the other sectors and not supplying to any of them, behaving then as a final consumer. The rest of the setup behaves as in the general framework, assuming that the household account for final demand in consumption, investment and government expenditures.

We use the following production function where firms producing good  $i$  use labor, capital and intermediate goods :

$$\begin{aligned} \max \pi_i &= p_i y_i - w_i l_i - r k_i - \sum_j p_j x_{ij} \\ \text{s.t. } y_i &= \left( \omega_{il}^{1/\sigma_y} (z_{iL} l_i)^{\frac{\sigma_y-1}{\sigma_y}} + \omega_{ik}^{1/\sigma_y} (k_i)^{\frac{\sigma_y-1}{\sigma_y}} + \sum_j \omega_{ij}^{1/\sigma_y} x_{ij}^{\frac{\sigma_y-1}{\sigma_y}} \right)^{\frac{\sigma_y}{1-\sigma_y}} \end{aligned}$$

The decision-making of the final demand consumers will look as follow :

$$\max \mathcal{U}_{FD} = \left( \omega_{ci}^{1/\sigma_c} c_i^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c-1}} \quad \text{s.t.} \quad \sum_i w_i l_i + \sum_i r k_i = \sum_i p_i c_i$$

The market clearing conditions are as follow :

$$y_i = c_i + \sum_j x_{ji}; \quad L = \sum_i l_i; \quad K = \sum_i k_i$$

**Calibration** Several ranges of parameters are necessary in order to solve this model in changes. We need the  $\Omega$  matrices of parameters  $\Omega_X$ ,  $\Omega_C$ ,  $\Omega_L$  and  $\Omega_K$  that we compute using the data from the federal planning bureau. We also need the Domar weights and the Leontief multipliers that we compute using the  $\Omega_X$  matrix for the Leontief multipliers and the data on gross output from the federal planning bureau and nominal GDP from the NBB national accounts for the Domar weights.

The technical coefficient matrix  $\Omega_X$  and the Leontief multipliers matrix  $\Psi$  correspond to the following graphical representations in [figure 14](#) and [figure 15](#).<sup>2</sup>

<sup>2</sup>In these graphs, we do not include the imports and exports sectors which are disproportionately big compared to the others. We nevertheless present the graphs with these sectors included in the appendix.

In [figure 14](#), we have the importance of the supplying sectors (columns) as a share of the cost of the using sectors (rows). We see some recurring patterns appear and some important sectors emerge. We have that generally, sectors tend to have a large share of their cost originating from this same sector (the diagonal of the heatmap). We also observe that some sectors account for a large share of the cost of other sectors like sector 46 (wholesale trade, except of motor vehicles and motorcycles) or sector 69-70 (legal and accounting activities).

In [figure 15](#), we see a classical pattern of the input-output analysis appear. The large Leontief multipliers are concentrated only on the diagonal of the heatmap. This means that for each sector, the most important supplier, directly and indirectly is itself. To have a better idea of the off-diagonal patterns in the Belgian Leontief multiplier, we set the diagonal elements equal to zero in [figure 16](#).

This allows to see similar patterns as the ones observed in the technical coefficient matrix where several sectors are directly and directly dependent on sectors 46 (wholesale trade), 64 (financial services) or 69-70 (legal and accounting services).

We then compute the factor-use vectors  $\Omega_L$  and  $\Omega_K$  and the vector of Domar weights  $\lambda$  and  $\Lambda_L$ , which are shown in [figure 17](#). We see some patterns confirming what was in the NBB employment data. For example, sectors 84 (public administration), 85 (education) and 87-88 (residential care and social work activities) have a large share of their cost going to labor and the incomes of the workers in these sectors account for more than 10% of the total income in the economy (nominal GDP). These results also confirm the importance of some sectors through their Domar weights. Sectors such as sector 46, 64 and 69-70 all have a Domar weight higher than 0.1. There is also new information about the importance of capital cost for several sectors such as the sector 68 (real estate activities) and 77 (rental and leasing activities). And finally, we compute the final demand share of each sector  $\Omega_C$  for the joint final demand from households (consumption and investment) and the government, shown in [figure 18](#).

We have that some sectors are very important for household consumption such as sector 47 (retail trade), 55-56 (accommodation and food service activities), 68 (real estate) and 68a (imputed rents) and for investment such as sector 41-43 (construction). On the other hand, some sectors are very important for government spending such as sectors 84 (public administration and defense; compulsory social security), 85 (education) and 86 (human health activities).

Concerning the elasticities in our model, we borrow from the empirical results in the production network literature. For the elasticity between factors and intermediate goods, we use the results from [[Oberfield and Raval, 2021](#)] and [[Atalay, 2017](#)] for the elasticity between labor and capital and between intermediate goods. In [[Oberfield and Raval, 2021](#)], the authors find an elasticity between labor and capital in the range between 0.5 and 0.7. In [[Atalay, 2017](#)], the author finds an elasticity between intermediate goods consistently lower than 0.2. As we do not differentiate between these elasticities in our model, we take the lower bound of the range from [[Oberfield and Raval, 2021](#)] for the general elasticity of substitution in the production process for all sectors.

For the elasticity of substitution in final demand consumption, we use the estimates from [[Herrendorf et al., 2013](#)] and [[Oberfield and Raval, 2021](#)] for cross-industry elasticity of substitution ranging between 0.85 and 1 (all final demand sectors included).

sectoral elasticity	$\sigma_y$	0.5	[Oberfield and Raval, 2021]; [Atalay, 2017]
final demand elasticity	$\sigma_c$	0.9	[Herrendorf et al., 2013]; [Oberfield and Raval, 2021]
workers' mobility elasticity	$\kappa$	1.4	[Galle and Lorentzen, 2021]

Table 1: Calibration of the main elasticities

## 7 Results

In this section, we study two sets of results following labor-specific shocks in the energy sector and a shock to workers' mobility elasticity  $\kappa$ . For both sets of results, we study the first and second-order impact on GDP, the decomposition of sectoral wage elasticities by channel and descriptive statistics.

### 7.1 Factor specific productivity shocks

**Real GDP results** We start with the labor-specific productivity shock in the "electricity, gas, steam and air conditioning supply" sector. As described in [Baqae and Farhi, 2019], a shock to that sector implies non-linearities when we allow for shares to adjust at general equilibrium. The non-linearities exacerbate (dampen) the negative (positive) impact of the first-order effect. We make the simulations for different specifications in our comparative statics section. We use the following equation:

$$d \log \mathcal{Y} = \frac{d \log \mathcal{Y}}{d \log z_{EI}} d \log z_{EI} + \frac{1}{2} \frac{d^2 \log \mathcal{Y}}{d \log z_{EI}^2} (d \log z_{EI})^2$$

where  $\frac{d \log \mathcal{Y}}{d \log z_{EI}}$  and  $\frac{d^2 \log \mathcal{Y}}{d \log z_{EI}^2}$  are respectively the first and second-order elasticities of real GDP to a labor-specific shock in the energy sector. Computing these elasticities in our model, we have the results in table 2.

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d \log \mathcal{Y}}{d \log z_{EI}}$	$\frac{d^2 \log \mathcal{Y}}{d \log z_{EI}^2}$
(0.5, 0.9, 1.4)	0.005	-0.007

Table 2: Results of the real GDP elasticities to labor-specific productivity shocks

Table 2 implies that non-linearities exacerbate negative shocks and dampen positive shocks, as the second-order elasticity is negative. Then, we plot figure 5 for the impact on real GDP of a range of productivity shocks:

where the red line is the first-order impact of the shock as described by Hulten's theorem and the blue line is the impact of the shock up to a second-order approximation.

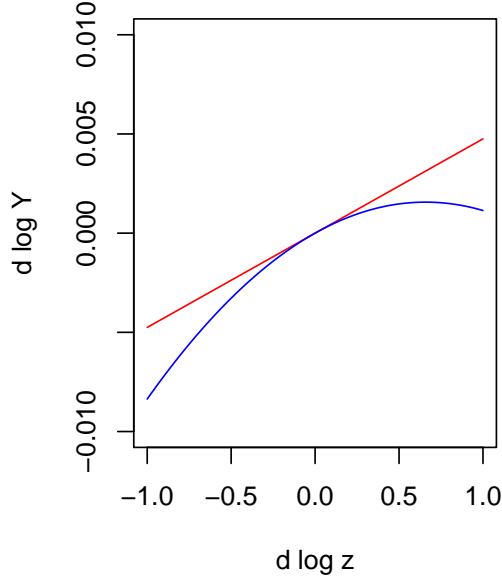


Figure 5: 1st and 2nd order effect of labor-specific productivity shock in the energy sector to real GDP

**Wage inequality** Second, studying how wages react in this model following the shock, we look at the specification of the system of wage elasticities in this model:

$$\begin{aligned} \frac{d \log w_i}{d \log z_{El}} &= \hat{\sigma}_{y,i} \left( \frac{d \log w_i}{d \log z_{El}} - \frac{d \log p_i}{d \log z_{El}} \right) + \frac{1}{\lambda_i} \hat{\sigma}_c \sum_o \Omega_{co} \Psi_{oi} \left( \frac{d \log p_o}{d \log z_{El}} - \frac{d \log \mathcal{P}}{d \log z_{El}} \right) \\ &+ \sum_k \frac{\lambda_k}{\lambda_i} \hat{\sigma}_{y,k} \sum_o \Omega_{ko} \Psi_{oi} \left( \frac{d \log p_o}{d \log z_{El}} - \frac{d \log p_k}{d \log z_{El}} \right) - \kappa \frac{\Phi_i}{l_i} \left( \frac{d \log w_i}{d \log z_{El}} - \sum_o \Phi_o \frac{d \log w_o}{d \log z_{El}} \right) + \Lambda_s \end{aligned}$$

where we recognize the general characterization above but specific to a labor shock in the energy sector. Solving it, we show the results for the mean and standard deviation of the wage elasticities in [table 3](#).

mean wage elasticities	s.d. wage elasticities	25th percentile	75th percentile
0.004	0.18	0.00006	0.04

Table 3: Results for the wage elasticities to productivity shocks

First, these results indicate that the mean impact is mild for a low level of productivity shock, which confirms the result of the impact for real GDP. Second, the standard devia-



tion indicates that there is a clear potential for changes in wage inequality in this setup. Taking the 25th and the 75th percentile, we have that the middle 50% of wage elasticities are between 0.00006 and 0.04. We plot on [figure 6](#) the results for wage elasticities following a labor-specific productivity shock in the energy sector.

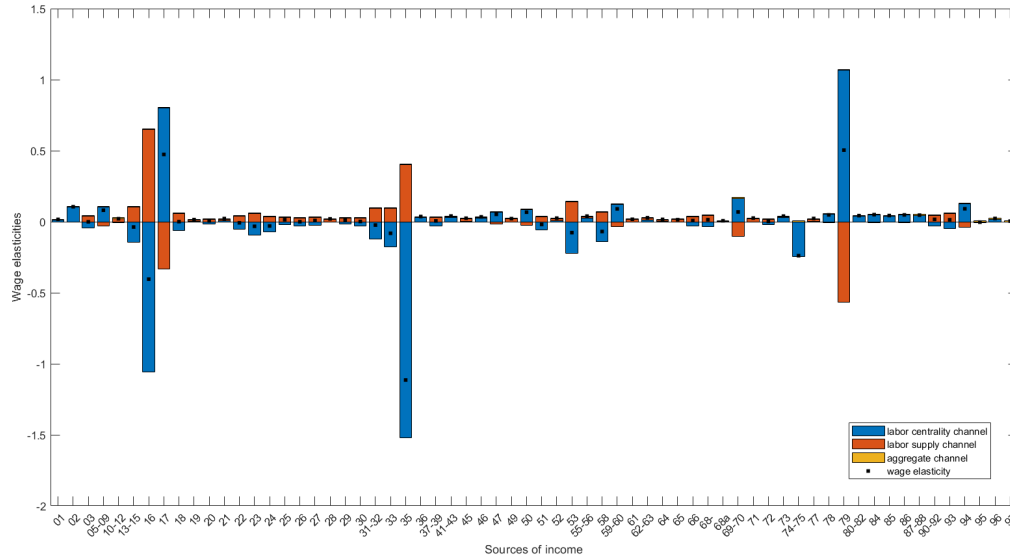


Figure 6: Decomposition of wage elasticities to productivity shocks

In [figure 6](#), we also decompose the wage elasticity by channel (see general characterization). Here, we have first that when we allow for changes in wages in our setup, the aggregate channel accounts for a very small share of the wage elasticities. Second, we have that the labor centrality and the labor supply channels always go in opposite directions and their difference determines the net wage elasticity. Take first the case when  $\kappa \rightarrow 0$ , we have that workers are stuck in their sector and the labor supply channel is always equal to zero. Hence we have that the wage elasticities will be the sum of the blue and yellow bars in the graph above. However, when workers are at least somewhat mobile, the change in labor centrality impacts wages which impacts workers' optimal decision of where to supply their labor. For example, take a shock that increases labor centrality in sector  $i$ , then the demand for labor in that sector increases directly and indirectly through the production network which increases the equilibrium wage in sector  $i$ . However, as the wage in sector  $i$  becomes more interesting compared to the other sectors, workers move to that sector which increases labor supply and decreases equilibrium wage in sector  $i$ . This is the net impact of these two channels, to which we add the aggregate effect, which determines the equilibrium wage elasticity.

In the other extreme case, when workers are perfectly mobile, as  $\kappa \rightarrow \infty$ , wage elasticity cannot differ from zero as any small change in comparative wage implies a disproportionate change in labor supply which impedes any wage change.

**Comparative statics** Finally, we proceed to some comparative statics with different calibrations for  $\sigma$  and  $\kappa$  in [table 4](#). For the different characterization of  $\sigma$ , we have that the

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d \log \mathcal{Y}}{d \log z_{EI}}$	$\frac{d^2 \log \mathcal{Y}}{d \log z_{EI}^2}$	mean wage elasticities	s.d. wage elasticities
(0.5, 0.9, 1.4)	0.005	-0.007	0.004	0.18
(0.1, 0.9, 1.4)	0.005	0.0008	0.002	9.39
(0.5, 0.9, 1.1)	0.005	-0.007	0.004	0.85
(0.5, 0.9, 2)	0.005	-0.007	-0.06	0.73

Table 4: Results of the real GDP elasticities to workers' mobility shocks

second-order elasticity  $\frac{d^2 \log \mathcal{Y}}{d \log z_{EI}^2}$  and the dispersion of wage elasticities increase as the elasticity of substitution between factors and intermediate goods get closer to zero.

Second, for different levels of workers' mobility, we have that as the elasticity  $\kappa$  increases, and therefore workers are more mobile between sectors, the wage elasticities become less dispersed and concentrate around a decrease mean wage elasticity.

## 7.2 Shock to workers' mobility

**Real GDP results** In this second exercise, we study the impact of a change in workers' mobility elasticity  $\kappa$  on real GDP up to a second-order approximation and its impact on the dispersion of wages. Start with the impact on real GDP, using the new equations that we developed to estimate first and second-order impact of exogenous changes in  $\kappa$ , we present the first-order and second-order elasticities in [table 5](#).

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d \log \mathcal{Y}}{d \log \kappa}$	$\frac{d^2 \log \mathcal{Y}}{d \log \kappa^2}$
(0.5, 0.9, 1.4)	0.14	-0.34

Table 5: Results of the real GDP elasticities to workers' mobility shocks

where we have that the first-order impact of an increase in workers' mobility on real GDP is positive, which we would expect if workers' preferences and wages at the initial equilibrium are not aligned with sectors' importance for final demand  $\Lambda$ . We also have that the second-order effect is negative which implies that, as for the non-linearities for factor-specific productivity shocks, negative shocks are amplified by the non-linearities

while positive shocks to  $\kappa$  are dampened. Once introduced in the following equation, we are able to plot these first and second-order elasticities of real GDP  $\mathcal{Y}$ :

$$d \log \mathcal{Y} = \frac{d \log \mathcal{Y}}{d \log \kappa} d \log \kappa + \frac{1}{2} \frac{d^2 \log \mathcal{Y}}{d \log \kappa^2} (d \log \kappa)^2$$

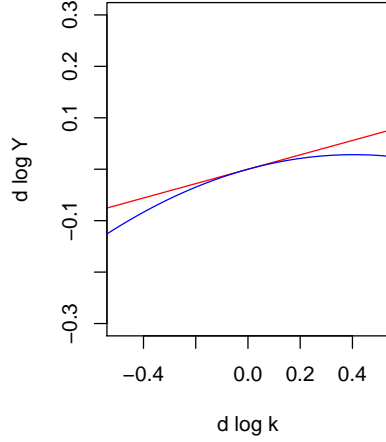


Figure 7: 1st and 2nd order effect of workers' mobility shock to real GDP

However, it is not the case that the non-linearities for shocks to workers' mobility are always negative. The sign and size of these non-linearities also depends on how the initial equilibrium compares with the sectoral labor importance for final demand, i.e.  $\Lambda$ . We investigate these graphically in [figure 8](#), [figure 9](#) and [figure 10](#).

Now, take the second-order elasticities as described in the general characterization section:

$$\begin{aligned} \frac{d^2 \log \mathcal{Y}}{d \log \kappa^2} = & \underbrace{\kappa \sum_j \Lambda_j \left( \log w_j - \sum_o l_o \log w_o \right)}_{\text{scale effect of } \kappa \text{ change}} + \underbrace{\kappa \sum_j \frac{d \Lambda_j}{d \log \kappa} \left( \log w_j - \sum_o l_o \log w_o \right)}_{\text{change in final demand importance}} \\ & + \underbrace{\kappa \sum_j \Lambda_j \left( \frac{d \log w_j}{d \log \kappa} - \sum_o l_o \frac{d \log w_o}{d \log \kappa} \right)}_{\text{change in comparative wages}} \end{aligned}$$

It embodies three channels through which a change in workers' mobility impacts real GDP. First, the change in  $\kappa$  implies a scale effect that reflects the first-order effect. The sign and the size of this effect depends on the alignment of comparative wages  $\log w_j - \sum_l l_l \log w_l$  and the importance of each labor for final demand  $\Lambda_j$ . Here, we have that this first channel is positive which implies that wages are usually greater for labor that are

more important for final demand through the production network. Hence, an increase in workers' mobility would increase labor supply in sectors that are more important as a labor supplier for final demand. This would therefore increase real GDP which implies a positive effect. In [figure 8](#), we see the intuition that comparative wage tend to be positive where incomes shares  $\Lambda_i$  are greater:

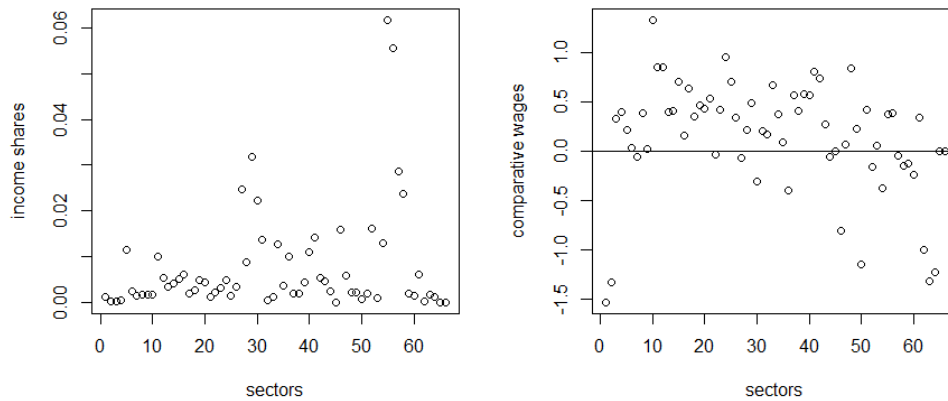


Figure 8: Income shares and comparative wages

The two other effects impacting the second-order elasticities of real GDP to  $\kappa$  shocks represent the endogenous change between income shares and comparative wages. It is positive when the endogenous reallocation of resources in the production network re-enforce the alignment of comparative wages and the importance of sectoral labor for final demand. On the other hand, when the reallocation of resources in the network disperse comparative wages with respect to income shares, the positive shocks are dampened and the second-order elasticity of real GDP is negative.

Consider [figure 9](#) and [figure 10](#) representing respectively how change in income shares move towards or opposite to the comparative wages and how comparative wages move towards or opposite to income shares.

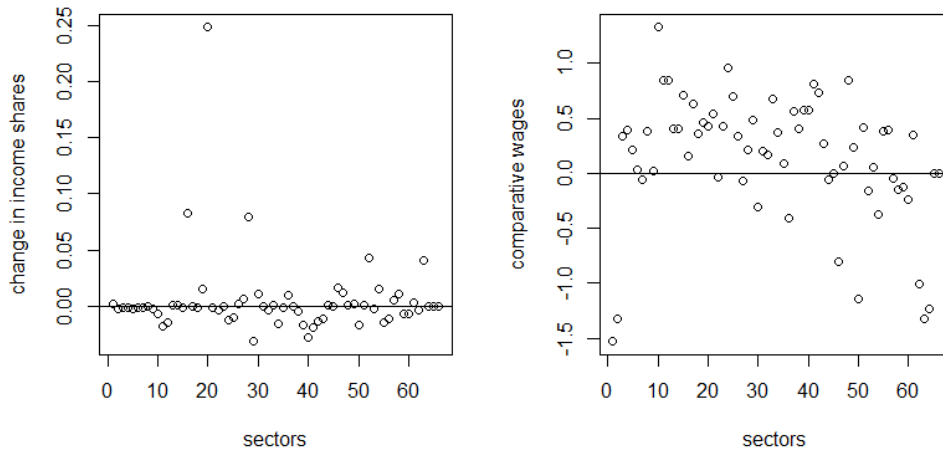


Figure 9: Change in sales shares and comparative wages

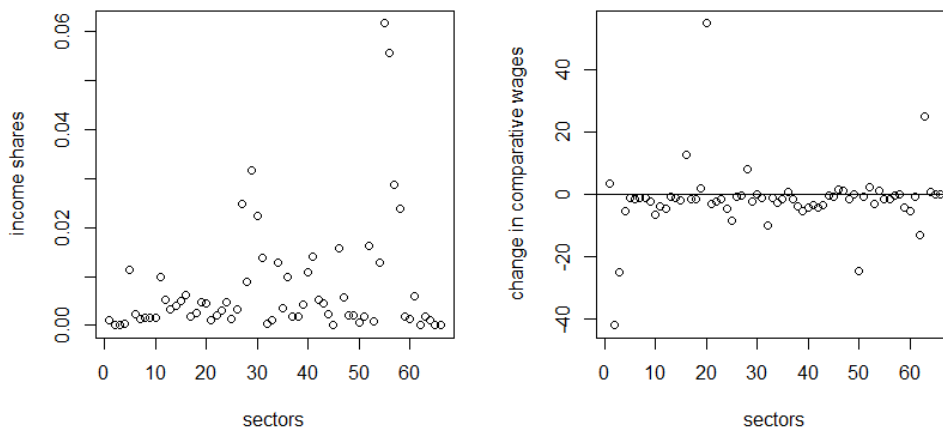


Figure 10: Change in comparative wages and income shares

**Wage inequality** Now, we study the dispersion of wage elasticities following the shock to workers' mobility. Take the following expression for wage elasticities:

$$\begin{aligned}
\frac{d \log w_i}{d \log \kappa} = & \underbrace{\hat{\sigma}_{y,i} \left( \frac{d \log w_i}{d \log \kappa} - \frac{d \log p_i}{d \log \kappa} \right) + \frac{1}{\lambda_i} \hat{\sigma}_c \sum_o \Omega_{co} \Psi_{oi} \left( \frac{d \log p_o}{d \log \kappa} - \frac{d \log \mathcal{P}}{d \log \kappa} \right)}_{\text{labor centrality channel}} \\
& + \underbrace{\sum_k \frac{\lambda_k}{\lambda_i} \hat{\sigma}_{y,k} \sum_o \Omega_{ko} \Psi_{oi} \left( \frac{d \log p_o}{d \log \kappa} - \frac{d \log p_k}{d \log \kappa} \right)}_{\text{labor centrality channel}} \\
& - \underbrace{\kappa \left( \log w_i - \sum_o l_o \log w_o \right)}_{\text{labor supply channel}} + \underbrace{\left( \kappa \sum_j \Lambda_j \left( \log w_j - \sum_o l_o \log w_o \right) \right)}_{\text{aggregate channel}}
\end{aligned}$$

where we have that how labor supply react to the shock differs from the productivity shock. In fact, here the change in wages do not matter for changes in labor supply up to a second-order approximation of change in real GDP as this the workers' mobility which is shocked and not the productivity of workers. Therefore, the labor supply is now impacted by the shock accordingly to how comparative wages were before the shock. Hence, if the wage in sector  $i$  was better compared to other wages before the shock, the change in workers' mobility will come exacerbate or dampen this effect on labor supply conditional on the shock being positive or negative.

Therefore, and as we observe in the decomposition of the shock below, the labor supply channel goes in the same direction as the labor centrality channel which exacerbates the wage elasticities to the shock.

In our results, we also have that the dispersion of wage elasticities is an order of magnitude greater than the one presented for the shock to productivity. This can be explained by the fact that labor supply effect come exacerbate labor centrality effects but it is also due to the use of standard deviation which is very sensitive to outliers. For the wage decomposition in [figure 11](#), we remove these outliers to make our results more readable but some sectoral wages experience disproportionately large change in wages which are not realistic. These workers account for 5.3% of the total workforce and we present the results in [figure 21](#).

mean wage elasticities	s.d. wage elasticities	25th percentile	75th percentile
-0.82	10.58	- 2.74	0.39

Table 6: Results for the wage elasticities to workers' mobility shocks

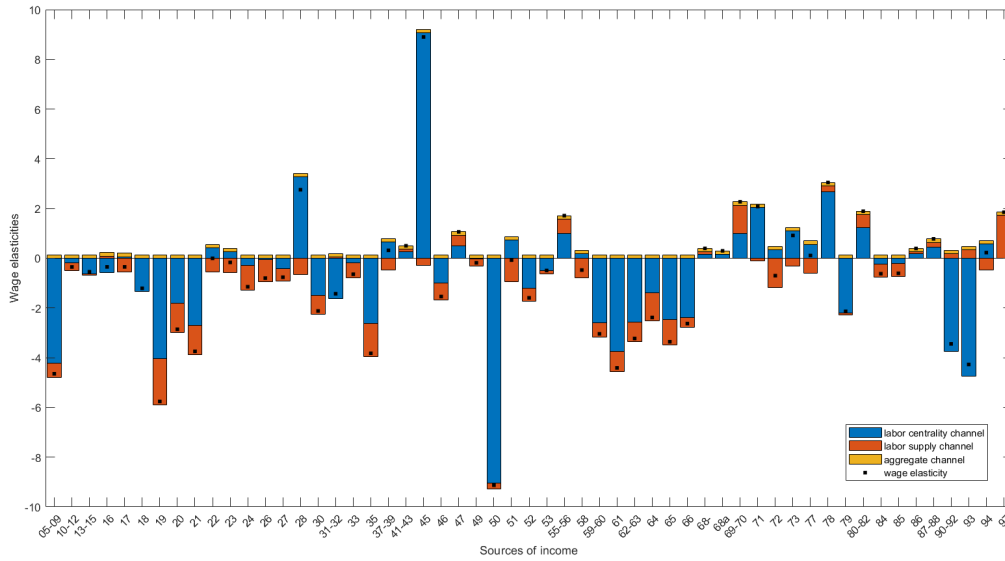


Figure 11: Decomposition of wage elasticities to shocks to  $\kappa$  (without outliers with one channel greater than 20 in absolute value)

**Comparative statics** Finally, we present some comparative statics for different calibrations in [table 7](#).

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d \log \mathcal{Y}}{d \log \kappa}$	$\frac{d^2 \log \mathcal{Y}}{d \log \kappa^2}$	mean wage elasticities	s.d. wage elasticities
(0.5, 0.9, 1.4)	0.14	0.19	-1.57	26.45
(0.1, 0.9, 1.4)	0.14	-0.69	-0.23	16.78
(0.5, 0.9, 1.1)	0.11	-0.19	-0.64	8.31
(0.5, 0.9, 2)	0.20	-0.79	-1.17	15.12

Table 7: Results of the real GDP elasticities to workers' mobility shocks

Here we look at the same specifications as the ones described for the factor-specific productivity shocks. First, keeping the elasticity between final goods and the elasticity of workers' mobility as fixed and varying the elasticity between factors and intermediate goods between 0.5 and 0.1, we have that as  $\sigma$  decreases, the second-order elasticity, the mean and the dispersion between wage elasticities increase.

Second, keeping the elasticity of substitution between factors and intermediate goods and between final demand goods as constant and varying the workers' mobility elasticity, we have that as  $\kappa$  increases, the second-order elasticity, the mean and the dispersion of wage

elasticities increase.

## 8 Conclusion

In this paper, we study the impact of labor-sector-specific productivity shocks on income inequality. Due to the existence of production networks, and the fact that workers are also consumers in the economy, productivity shocks can have general equilibrium effects throughout the economy. The magnitude of the inequality results depend on (i) the amount of labor mobility, and (ii) the convexity/concavity of the economy. When evaluating the impact of a change of labor mobility, we find strong effects on welfare and income inequality. Up to first order, a 1% increase in labor mobility generates a 0.24% increase in welfare. The second-order effects however show strong concavity. For low levels of mobility, providing slack on mobility has strong positive effects on welfare. In terms of wage inequality, we find strong and divergent effects across sectors. These results suggest that labor market improving policies such as training, support and education can have sizable welfare effects, while increasing/reducing income inequality, depending on the parameters of the model.



## References

- [Acemoglu and Autor, 2011] Acemoglu, D. and Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier.
- [Atalay, 2017] Atalay, E. (2017). How important are sectoral shocks? *American Economic Journal: Macroeconomics*, 9(4):254–80.
- [Autor et al., 2003] Autor, D. H., Levy, F., and Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly journal of economics*, 118(4):1279–1333.
- [Baqae and Farhi, 2019] Baqae, D. R. and Farhi, E. (2019). The macroeconomic impact of microeconomic shocks: beyond hulten’s theorem. *Econometrica*, 87(4):1155–1203.
- [Galle and Lorentzen, 2021] Galle, S. and Lorentzen, L. (2021). The unequal effects of trade and automation across local labor markets. *Available at SSRN 3800962*.
- [Herrendorf et al., 2013] Herrendorf, B., Rogerson, R., and Valentinyi, A. (2013). Two perspectives on preferences and structural transformation. *American Economic Review*, 103(7):2752–89.
- [Lagakos and Waugh, 2013] Lagakos, D. and Waugh, M. E. (2013). Selection, agriculture, and cross-country productivity differences. *American Economic Review*, 103(2):948–80.
- [Oberfield and Raval, 2021] Oberfield, E. and Raval, D. (2021). Micro data and macro technology. *Econometrica*, 89(2):703–732.

## Tables

NACE REV. 2	sector names
01	Crop and animal production, hunting and related service activities
02	Forestry and logging
03	Fishing and aquaculture
05_09	Mining and quarrying
10_12	Manufacture of food products, beverages and tobacco
13_15	Manufacture of textiles, wearing apparel, leather and related products
16	Manufacture of wood and of products of wood and cork, except furniture; articles of straw and plaiting materials
17	Manufacture of paper and paper products
18	Printing and reproduction of recorded media
19	Manufacture of coke and refined petroleum products
20	Manufacture of chemicals and chemical products
21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
22	Manufacture of rubber and plastics products
23	Manufacture of other non-metallic mineral products
24	Manufacture of basic metals
25	Manufacture of fabricated metal products, except machinery and equipment
26	Manufacture of computer, electronic and optical products
27	Manufacture of electrical equipment
28	Manufacture of machinery and equipment n.e.c.
29	Manufacture of motor vehicles, trailers and semi-trailers
30	Manufacture of other transport equipment
31_32	Manufacture of furniture, other manufacturing
33	Repair and installation of machinery and equipment
35	Electricity, gas, steam and air conditioning supply
36	Water collection, treatment and supply
37_39	Sewerage, waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
41_43	Construction
45	Wholesale and retail trade and repair of motor vehicles and motorcycles
46	Wholesale trade, except of motor vehicles and motorcycles
47	Retail trade, except of motor vehicles and motorcycles
49	Land transport and transport via pipelines
50	Water transport

NACE REV. 2	sector names
51	Air transport
52	Warehousing and support activities for transportation
53	Postal and courier activities
55_56	Accommodation and food service activities
58	Publishing activities
59_60	Audiovisual and broadcasting activities
61	Telecommunications
62_63	IT and other information services
64	Financial service activities, except insurance and pension funding
65	Insurance, reinsurance and pension funding, except compulsory social security
66	Activities auxiliary to financial service and insurance activities
68_	Real estate activities (excluding imputed rents)
68a	Imputed rents
69_70	Legal and accounting activities; activities of head offices; management consultancy activities
71	Architectural and engineering activities; technical testing and analysis
72	Scientific research and development
73	Advertising and market research
74_75	Other professional, scientific and technical activities; veterinary activities
77	Rental and leasing activities
78	Employment activities
79	Travel agency, tour operator, reservation service and related activities
80_82	Security and investigation activities; services to buildings and landscape activities; office administrative, office support and other business support activities
84	Public administration and defence; compulsory social security
85	Education
86	Human health activities
87_88	Residential care and social work activities
90_92	Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities
93	Sports activities and amusement and recreation activities
94	Activities of membership organizations
95	Repair of computers and personal and household goods
96	Other personal service activities
97	Activities of households as employers

# Figures

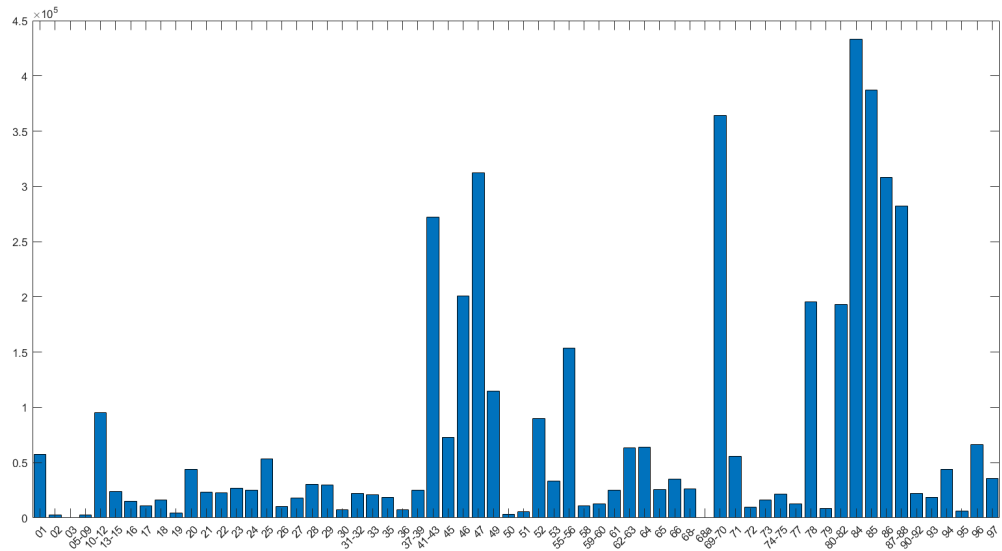


Figure 12: Labor allocation in the Belgian economy

Back to → [main paper](#)

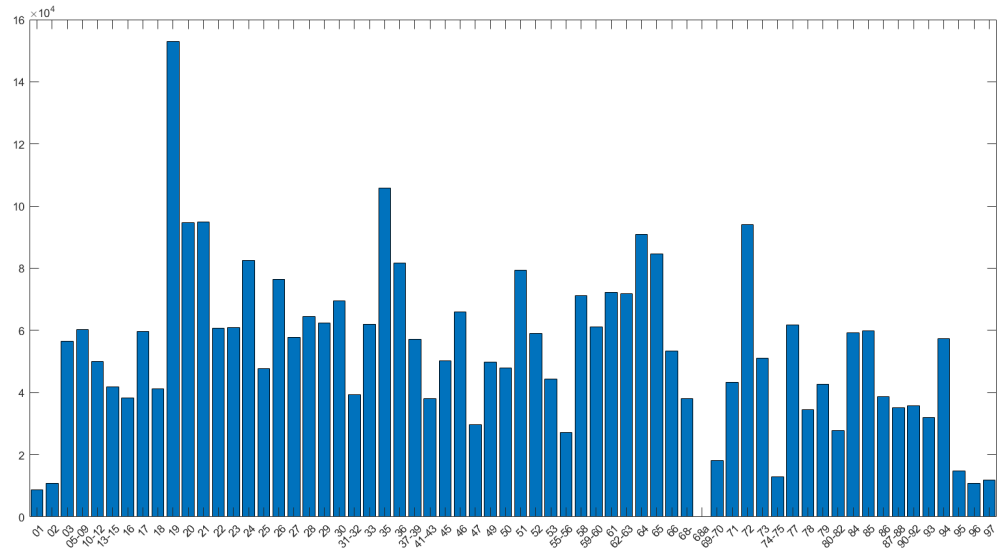


Figure 13: Sectoral mean annual wage

Back to → [main paper](#)

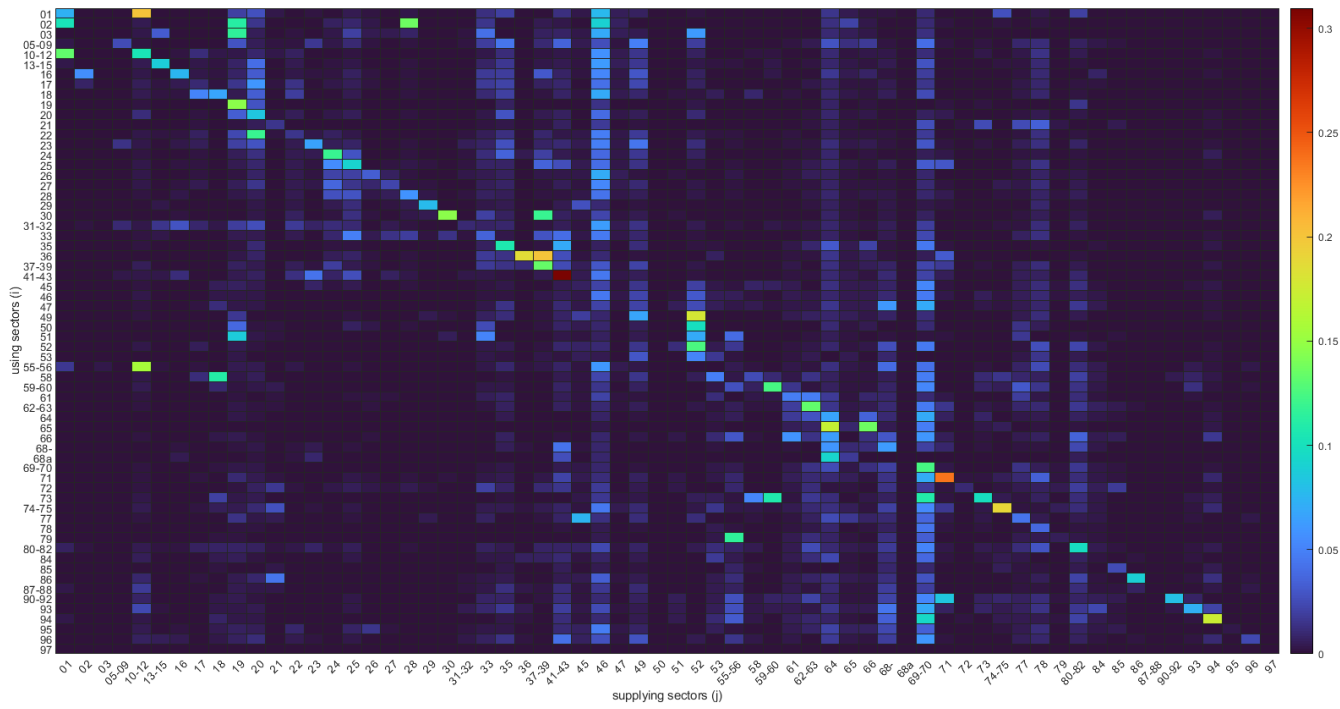


Figure 14: Technical coefficient matrix  $\Omega_X$

Back to → [main paper](#)

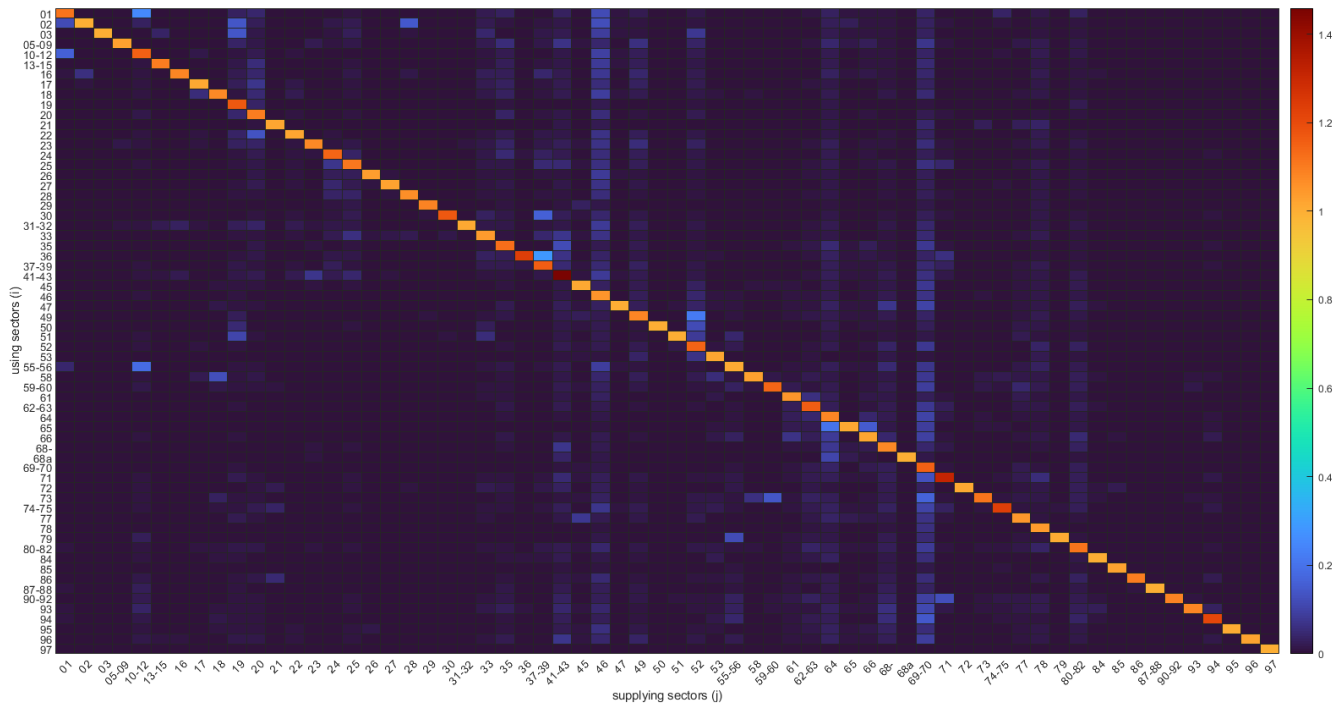


Figure 15: Leontief multipliers matrix  $\Psi$

Back to → [main paper](#)

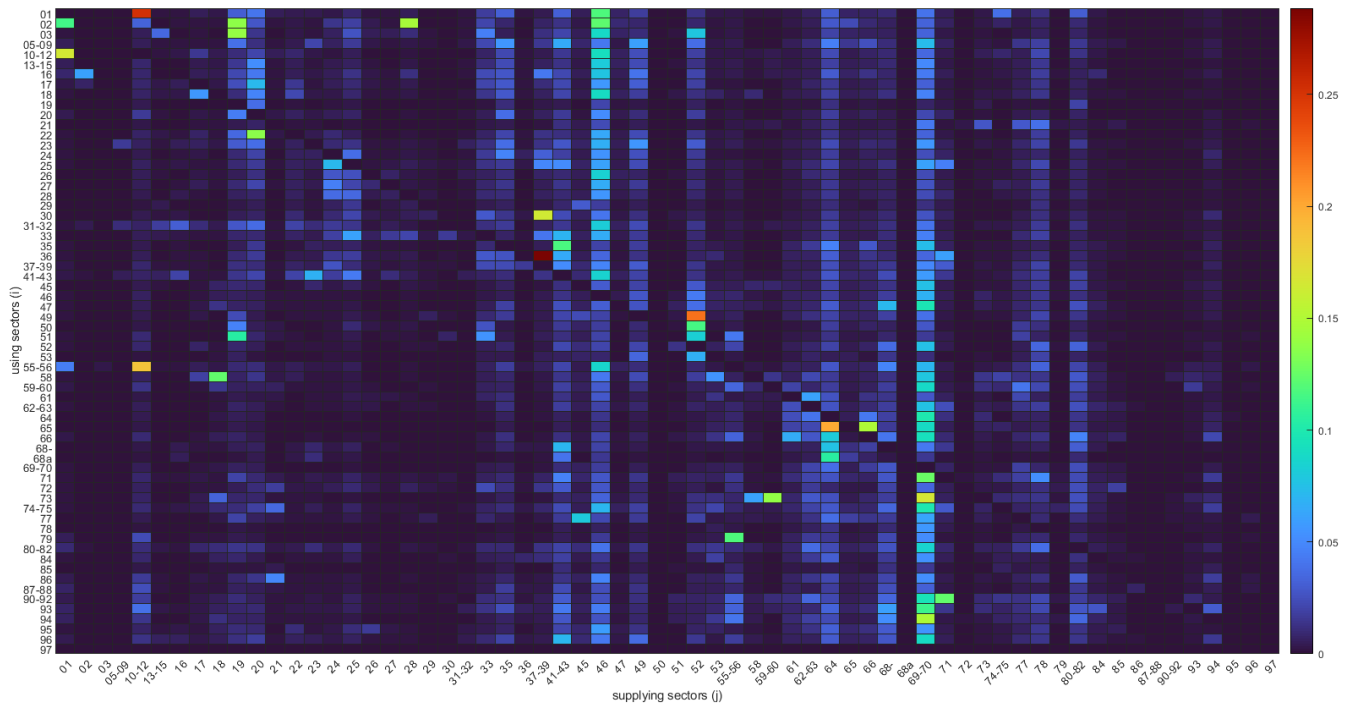


Figure 16: Off-diagonal elements of the Leontief multipliers matrix  $\Psi$

Back to → [main paper](#)



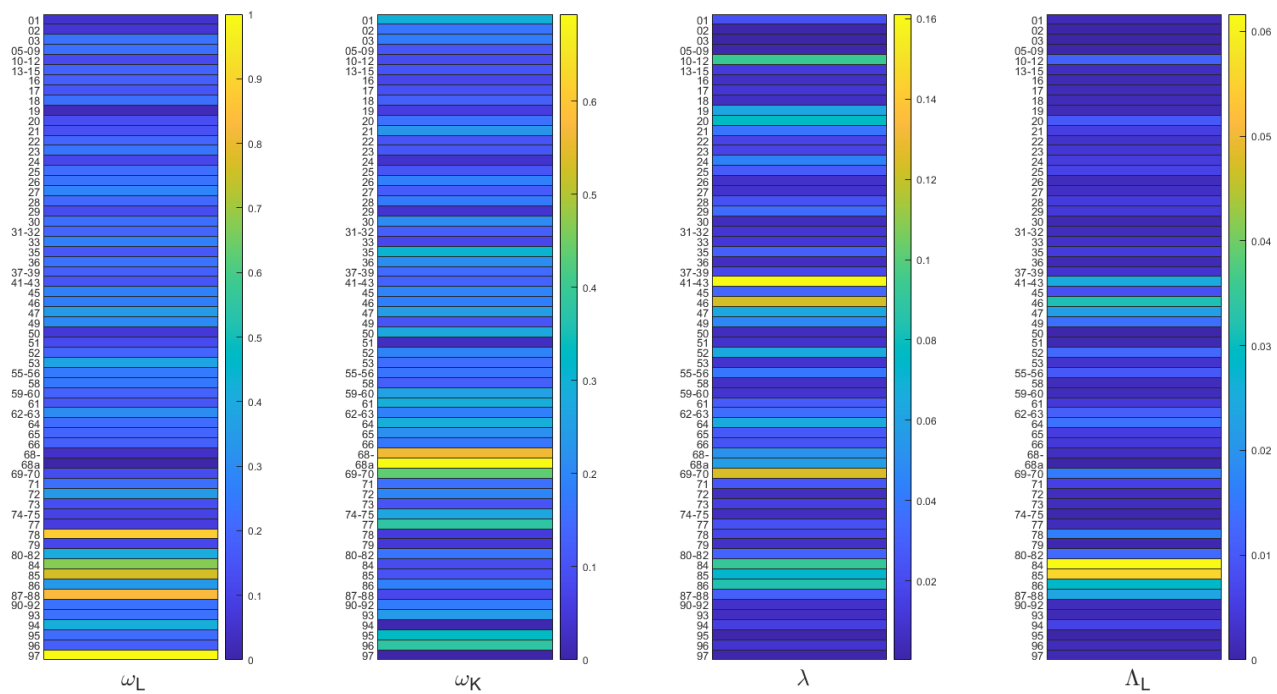


Figure 17: Labor share, capital and Domar weights by sector

Back to → [main paper](#)

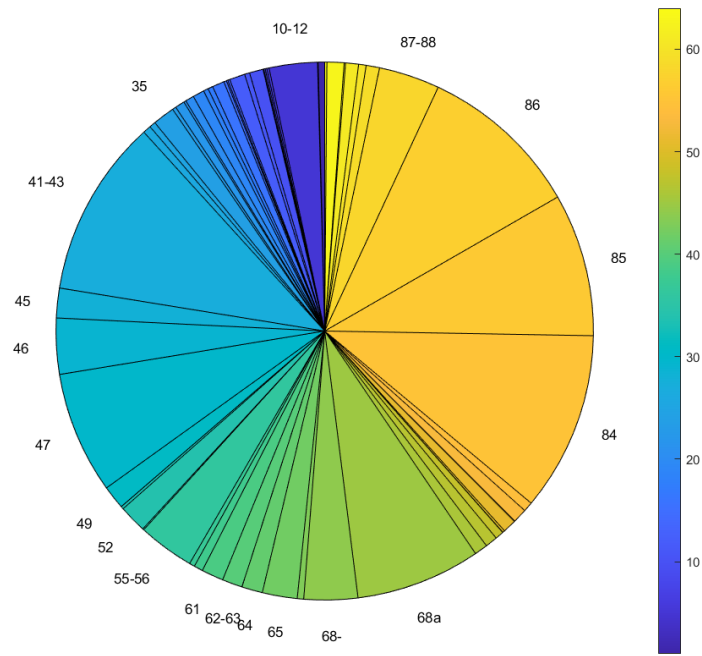


Figure 18: Consumption share from final demand

Back to → [main paper](#)

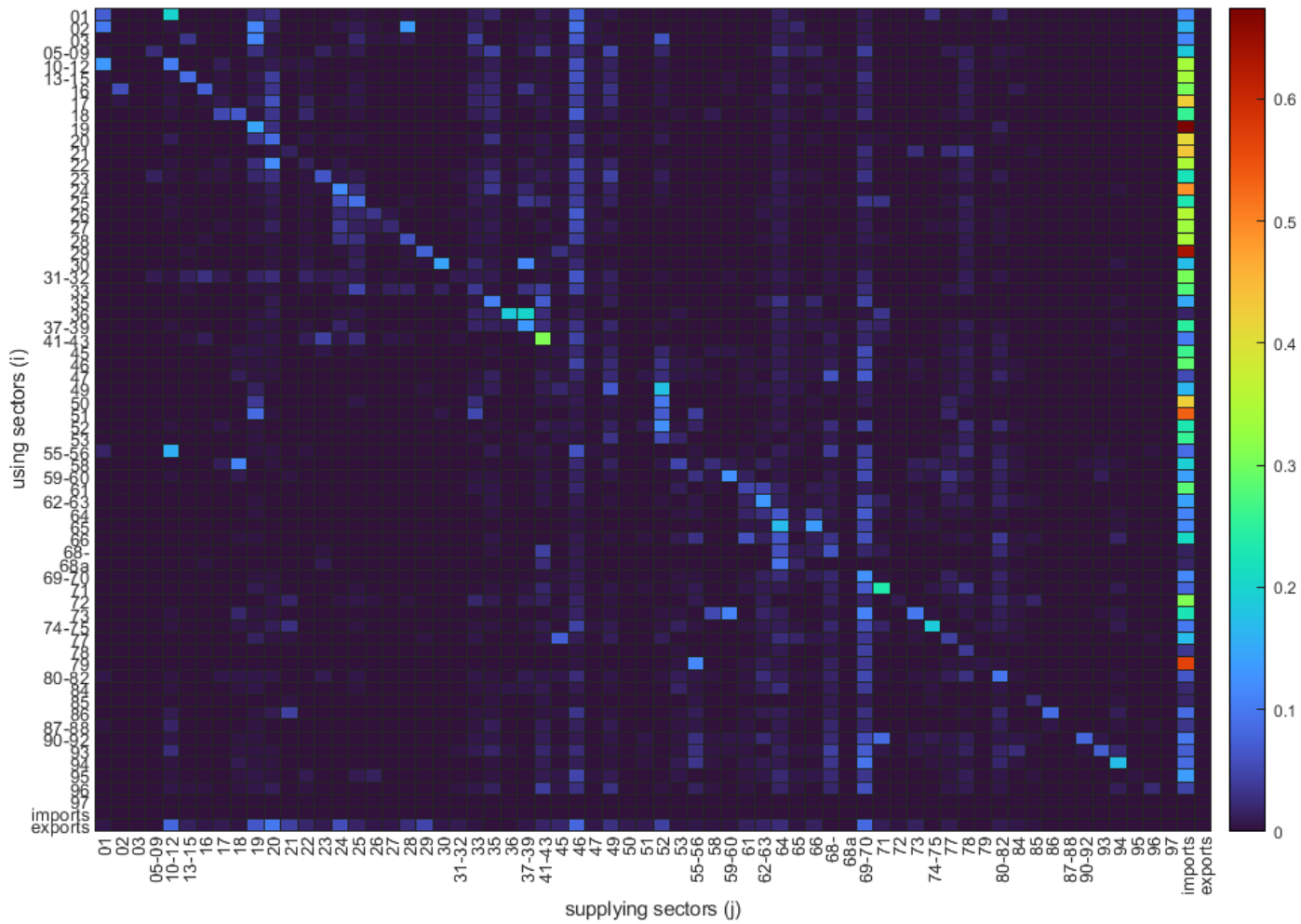


Figure 19: Matrix  $\Omega_X$  with imports and exports sectors

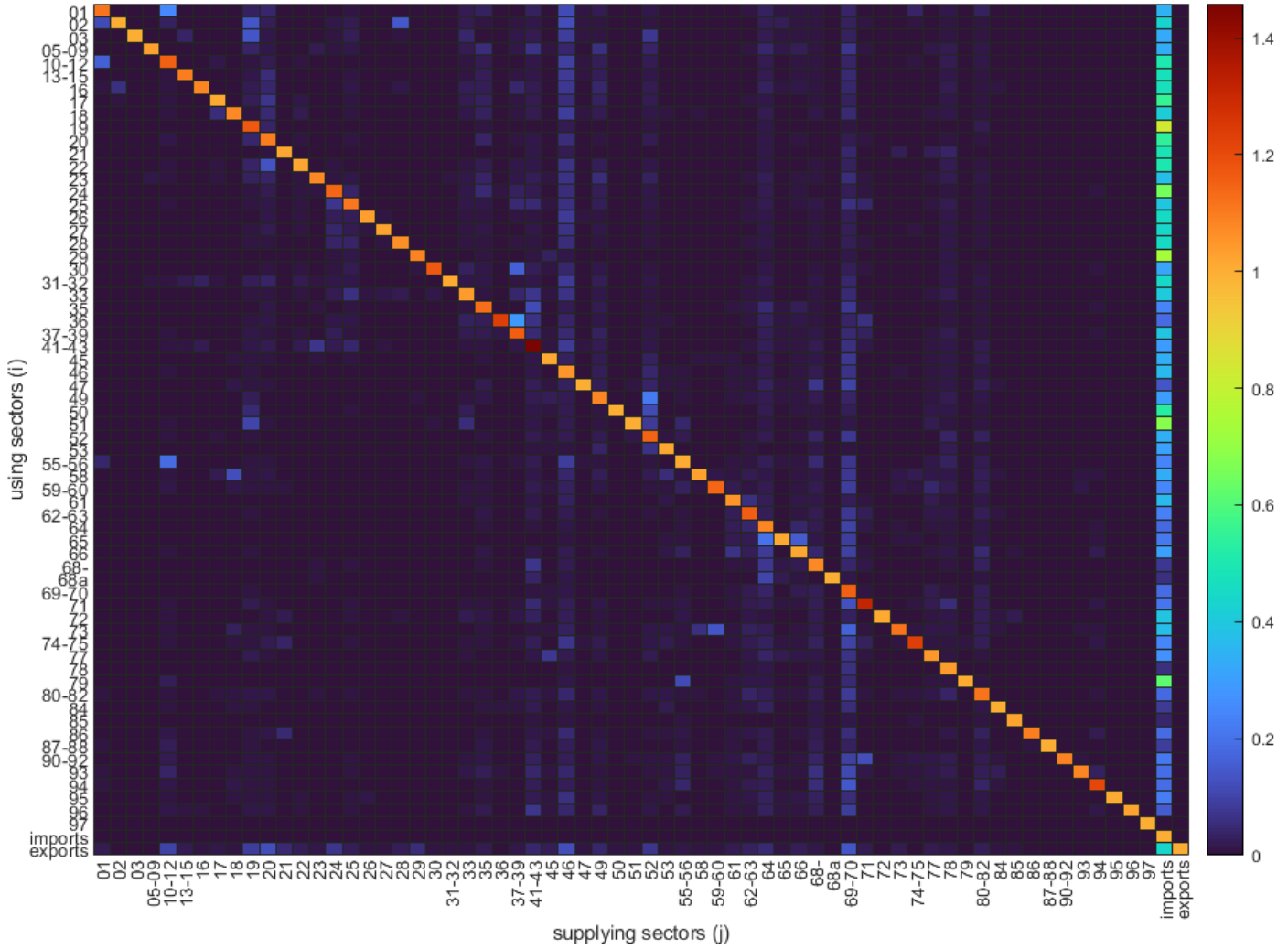


Figure 20: Matrix  $\Psi$  with imports and exports sectors

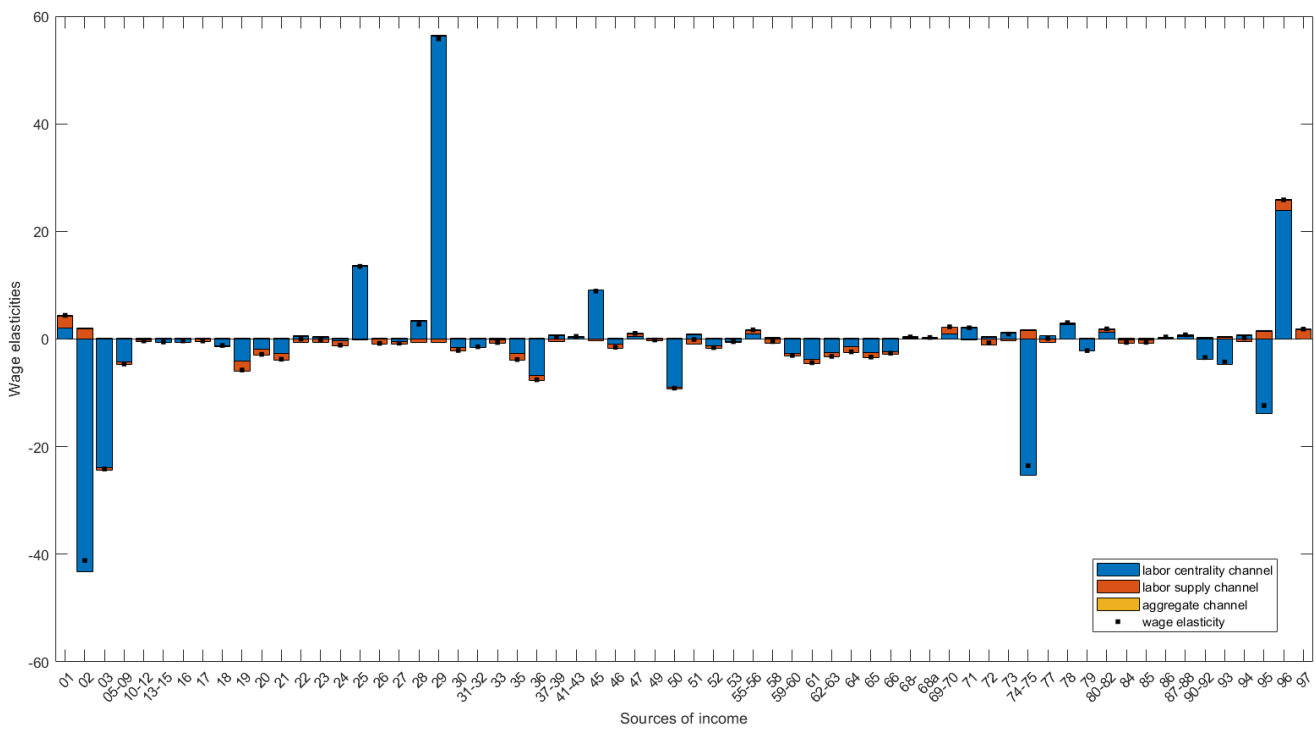


Figure 21: Wage elasticities to workers' mobility shocks

## NATIONAL BANK OF BELGIUM - WORKING PAPERS SERIES

The Working Papers are available on the website of the Bank: <http://www.nbb.be>.

381. "The heterogeneous employment outcomes of first- and second-generation immigrants in Belgium", by C. Piton and F. Rycx, *Research series*, January 2020.
382. "A Dane in the making of European Monetary Union – A conversation with Niels Thygesen", by I. Maes and S. Péters, *Research series*, May 2020.
383. "Multi-product exporters: Costs, prices and markups on foreign vs domestic markets", by Catherine Fuss, *Research series*, June 2020.
384. "Economic importance of the Belgian maritime and inland ports – Report 2018", by I. Rubbrecht and K. Burggraeve, *Document series*, July 2020.
385. "Service characteristics and the choice between exports and FDI: Evidence from Belgian firms", by L. Sleuwaegen and P.M. Smith, *Research series*, July 2020.
386. "Low pass-through and high spillovers in NOEM: What does help and what does not", by G. de Walque, T. Lejeune, A. Rannenberg and R. Wouters, *Research series*, July 2020.
387. "Minimum wages and wage compression in Belgian industries", by S. Vandekerckhove, S. Desiere and K. Lenaerts, *Research series*, July 2020.
388. "Network effects and research collaborations", by D. Essers, F. Grigoli and E. Pugacheva, *Research series*, July 2020.
389. "The political economy of financing climate policy – evidence from the solar PV subsidy programs", by O. De Groot, A. Gautier and F. Verboven, *Research series*, October 2020.
390. "Going green by putting a price on pollution: Firm-level evidence from the EU", by O. De Jonghe, K. Mulier and G. Schepens, *Research series*, October 2020.
391. "Banking barriers to the green economy", by H. Degryse, T. Roukny and J. Tielens, *Research series*, October 2020.
392. "When green meets green", by H. Degryse, R. Goncharenko, C. Theunisz and T. Vadasz, *Research series*, October 2020.
393. "Optimal climate policy in the face of tipping points and asset stranding", by E. Campiglio, S. Dietz and F. Venmans, *Research series*, October 2020.
394. "Are green bonds different from ordinary bonds? A statistical and quantitative point of view", by C. Ma, W. Schoutens, J. Beirlant, J. De Spiegeleer, S. Höcht and R. Van Kleeck, *Research series*, October 2020.
395. "Climate change concerns and the performance of green versus brown stocks", by D. Ardia, K. Bluteau, K. Boudt and K. Inghelbrecht, *Research series*, October 2020.
396. "Daily news sentiment and monthly surveys: A mixed-frequency dynamic factor model for nowcasting consumer confidence", by A. Algaba, S. Borms, K. Boudt and B. Verbeken, *Research series*, February 2021.
397. "A bigger house at the cost of an empty fridge? The effect of households' indebtedness on their consumption: Micro-evidence using Belgian HFCS data", by Ph. Du Caju, G. Périlleux, F. Rycx and I. Tojerow, *Research series*, March 2021.
398. "Low interest rates and the distribution of household debt", by M. Emiris and F. Koulischer, *Research series*, March 2021.
399. "The interplay between green policy, electricity prices, financial constraints and jobs. Firm-level evidence", by G. Bijmens, J. Hutchinson, J. Konings and A. Saint Guilhem, *Research series*, April 2021.
400. "Economic importance of the Belgian maritime and inland ports – Report 2019", by I. Rubbrecht, E. Dhyne and C. Duprez, *Research series*, May 2021.
401. "The return on human (STEM) capital in Belgium", by G. Bijmens and E. Dhyne, *Research series*, July 2021.
402. "Unraveling industry, firm and host-region effects on export behaviors of international new ventures and established exporters", by I. Paeleman, S. A. Zahra and J. W. B. Lang, *Research series*, July 2021.
403. "When trust is not enough: Bank resolution, SPE, Ring-fencing and group support" by M. Dewatripont, M. Montigny and G. Nguyen, *Research series*, August 2021.
404. "Bank specialization and zombie lending", by O. De Jonghe, K. Mulier and I. Samarin, *Research series*, November 2021.
405. "Robert Triffin, Japan and the quest for Asian Monetary Union", I. Maes and I. Pasotti, *Research series*, February 2022.
406. "The impact of changes in dwelling characteristics and housing preferences on house price indices", by P. Reusens, F. Vastmans and S. Damen, *Research series*, May 2022.
407. "Economic importance of the Belgian maritime and inland ports – Report 2020", by I. Rubbrecht, *Research series*, May 2022.

408. "New facts on consumer price rigidity in the euro area", by E. Gautier, C. Conflitti, R. P. Faber, B. Fabo, L. Fadejeva, V. Jouvanceau, J. O. Menz, T. Messner, P. Petroulas, P. Roldan-Blanco, F. Rumler, S. Santoro, E. Wieland and H. Zimmer, *Research series*, June 2022.
409. "Optimal deficit-spending in a liquidity trap with long-term government debt", by Charles de Beaufort, *Research series*, July 2022.
410. "Losing prospective entitlement to unemployment benefits. Impact on educational attainment", by B. Cockx, K. Declercq and M. Dejemepe, *Research series*, July 2022.
411. "Integration policies and their effects on labour market outcomes and immigrant inflows", by C. Piton and I. Ruysen, *Research series*, September 2022.
412. "Foreign demand shocks to production networks: Firm responses and worker impacts", by E. Dhyne, A. K. Kikkawa, T. Komatsu, M. Mogstad and F. Tintelnot, *Research series*, September 2022.
413. "Economic research at central banks: Are central banks interested in the history of economic thought?", by I. Maes, *Research series*, September 2022.
414. "Softening the blow: Job retention schemes in the pandemic", by J. Mohimont, M. de Sola Perea and M.-D.Zachary, *Research series*, September 2022.
415. "The consumption response to labour income changes, by K. Boudt, K. Schoors, M. van den Heuvel and J. Weytjens, *Research series*, October 2022.
416. "Heterogeneous household responses to energy price shocks, by G. Peersman and J. Wauters, *Research series*, October 2022.
417. "Income inequality in general equilibrium", by B. Bernon, J. Konings and G. Magerman, *Research series*, October 2022.

National Bank of Belgium  
Limited liability company  
RLP Brussels – Company's number: 0203.201.340  
Registered office: boulevard de Berlaimont 14 – BE-1000 Brussels  
[www.nbb.be](http://www.nbb.be)

Editor

**Pierre Wunsch**

Governor of the National Bank of Belgium

© Illustrations: National Bank of Belgium

Layout: Analysis and Research Group  
Cover: NBB CM – Prepress & Image

Published in October 2022